

DIGITAL SIMULATION OF 16-QAM MODULATION

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Abstract: In this work shows the rules for QAM modulation - Quadrature Amplitude Modulation. For 16-QAM modulation are given different constellation diagrams. Shows an example of a 16-QAM modulation. Then shows a simulation of the demodulation. Calculations in Matlab.

Keywords: modulation, simulation, constellation diagram, quadrature amplitude modulation.

1 Introduction to QAM modulation

QAM modulation (*Quadrature Amplitude Modulation*), is a quadrature modulation of amplitude-phase. Is used to send digital data over radio channel used in DVB broadcasts. Input to the modulation of the form string bits. QAM modulation is a combination of amplitude modulation and phase modulation. The data are formed into two, three, four, etc. that correspond to both amplitude and phase. Are created according to the diagram of the constellation (*Constellation diagram*). QAM signal is a linear combination of two orthogonal waveforms (shifted in phase by $\pi/2$): cosine and sine [8, 12, 14].

There are 2^n combinations created from n bits. Grouping data input n bits per symbol gives so 2^n points constellation, which are often referred to as phasors, or complex vectors. Phasors related to these points can have different amplitude and/or phase values and, therefore, this type of modulation is called multi-level modulation, where the number of levels is equal to the number of constellation points [2]. For $n=4$ bits are grouped after 4 and we have to deal with the modulation 16-QAM.

Data in digital form is divided into two streams. Then, each stream is converted to an analog signal in the digital-to-analog converter. Analog signal can pass through a low-pass filter. In the next stage of one signal is multiplied by the carrier, and the second by the carrier shifted in phase by $\pi/2$. At the end of both modulation signals are summed and sent as QAM signal. Channels through which pass signals to I (in-phase) and Q (in quadrature). Analog signal in the channel and is multiplied by the cosine function, and an analog signal on channel Q is multiplied by sine function [8, 12, 14].

The process of creating signal describe mathematical formulas [8, 12, 14]:

$$S_i(t) = A_i \cos(\omega_0 t + \phi_i) \quad (1)$$

$$S_i(t) = A_i(\cos \phi_i \cos \omega_0 t - \sin \phi_i \sin \omega_0 t) \quad (2)$$

$$S_i(t) = a_i \cos \omega_0 t - b_i \sin \omega_0 t \quad (3)$$

where:

$$a_i(t) = A_i \cos \phi_i \quad (4)$$

$$b_i(t) = A_i \sin \phi_i \quad (5)$$

These formulas describe the equivalence of modulation amplitude and phase with the sum of passes of the shifted in phase by $\pi/2$ [8, 12]. From the formula (3) shows that for i -this group of n consecutive bits, the signal on the channel I has the form $a_i \cos(\omega_0 t)$, and the channel Q $-b_i \sin(\omega_0 t)$.

2 16-QAM constellation and 16-QAM modulation

As before I was told a group of n bits shall be assigned a point in the constellation.

Constellation diagram is a graphical representation of the modulated signal digitally, for example, using the QAM or PSK. It shows the signal as a two-dimensional graph on the complex plane. If the signal is represented by a complex number can be selected in the complex plane. The real axis is often called the axis of the in-phase (I), and the imaginary axis the quadrature axis (Q). Points for all modulation signals, make up the constellation diagram are called constellation points [9]. Each constellation point is assigned the amplitude and phase. Amplitude, the distance of the constellation point from the origin. Phase, is the angle between the axis of the actual I (horizontal), and the vector between the origin and the point of the constellation. Modulation 16-QAM can also be different layouts of the constellation. There are three types of constellation. They are shown in figure 1.

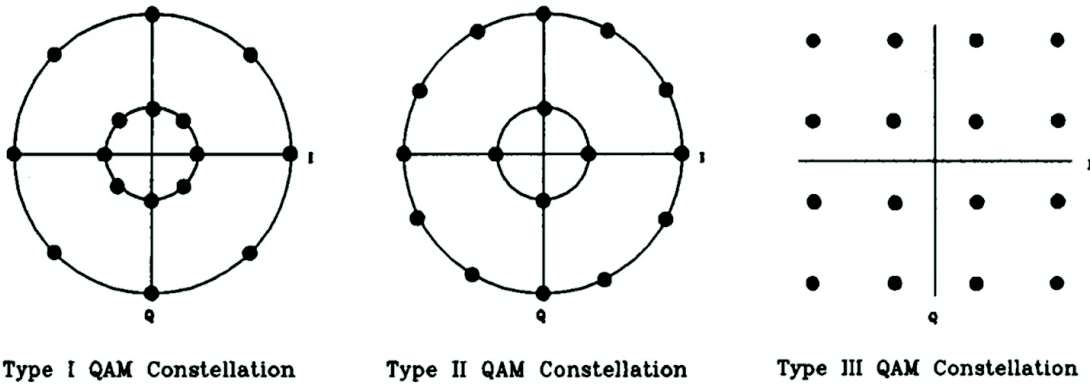
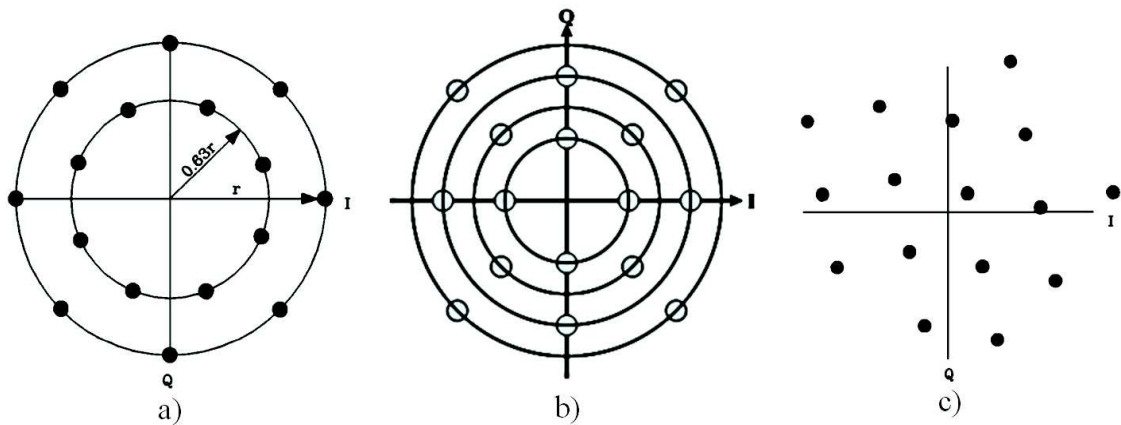


Fig. 1. Examples of the QAM constellation types I, II and III [2, 7]

There are also different variations of these types of constellation.



**Fig. 2. a) optimum 16 levels constellation by Lucky [6]
 b) not orthogonal 16-QAM modulation [10]
 c) optimum 16 levels constellation by Foschini [1]**

In practice, the greatest recognition met with constellations rectangular type III presented in figure 1. Other types of constellation does not apply due to the complexities associated with calculations.

You must assign the points of constellation 16 combinations of four bits. You can also do so in different ways. Important in the coding of the constellation points are to this encoding was using Gray code. Gray code is a binary code without the weight, not position, characterised by the fact that the next two code words differ only state one bit (in this case, two adjacent four bits to constellation differ only by a single bit). In other words, the Hamming Distance between adjacent four bits is 1.

Each point of the constellation (i.e. each four bits are assigned coordinates in the plane. Typically, it is assumed that the horizontal axis „I” and on the vertical axis „Q” the coordinates of the points are: -3, -1, 1 3. This is shown in the figure 3. It is important that distance in the horizontal and vertically between the points were the same.

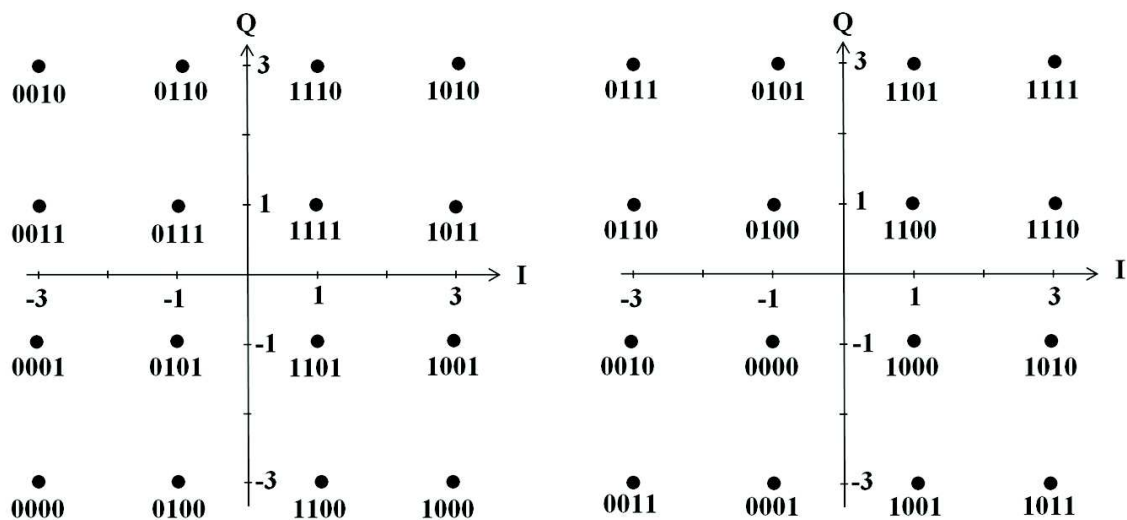


Fig. 3. Constellation diagrams for 16-QAM [13]

Other placement of four bits in the constellation can be seen in the work of [8, 9, 10, 11, 12, 15].

Constellation diagram can be represented as a matrix (6), a two-bit parts the coded numbers a_i (4), b_i (5), indicate the elements of the array that contains the amplitude I and Q component waves. The diagram assigns to the coded numbers amplitude I and Q waves. Exactly, it $|a_i|$ is the amplitude of the signal $\cos(\omega_0 t)$ in the channel I (3), and $|b_i|$ is the amplitude of the signal $-\sin(\omega_0 t)$ in channel Q (3).

$$[a_i, b_i] = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix} \quad (6)$$

Source: [8, 12]

So in 16-QAM modulation with a string of bits input groups after the four bits. For each four bits, based on selected constellations, specifies the number of a_i and b_i . The number of the digital-to-analog signals are converted into constant $a_i(t)$ (4) and $b_i(t)$ (5). Then in the channel and the signal $a_i(t)$ is multiplied by $\cos(\omega_0 t)$, and in the channel Q signal $b_i(t)$ is multiplied by the $-\sin(\omega_0 t)$. The resulting QAM signal is obtained by adding both the signals of the channels I and Q (3). This is shown in figure 4.

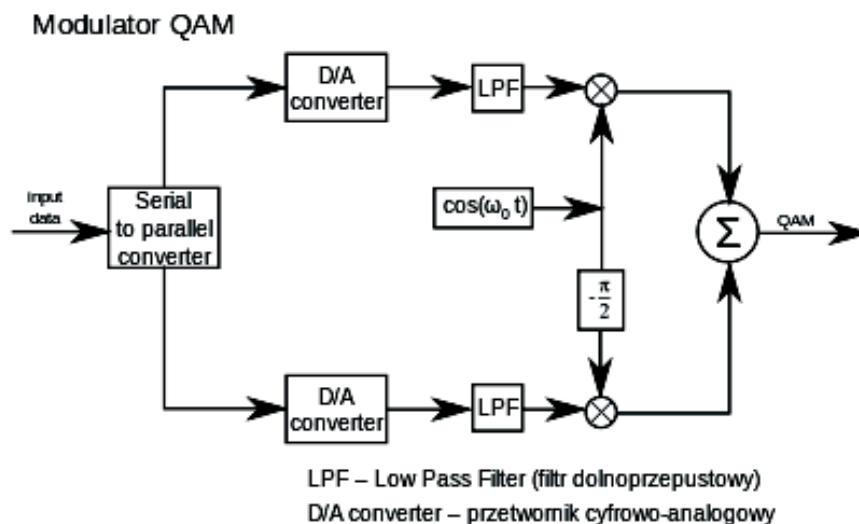


Fig. 4. Flowchart QAM modulation [8]

The above considerations we will illustrate the example.

3 Example 16-QAM modulation

Example 1

We have the following string input: $\{q\} = 10100100111100011000$. These data are grouped four bits: 1010 0100 1111 0001 1000. Select the constellation, for example. the first figure 3. Then we have $(a_i, b_i) = (3, 3), (-1, -3), (1, 1), (-3, -1), (3, -3)$.

Signals on the channel I will have the form (3):

$$a_i \cos(\omega_0 t) = 3\cos(\omega_0 t), -\cos(\omega_0 t), \cos(\omega_0 t), -3\cos(\omega_0 t), 3\cos(\omega_0 t).$$

Signals on the channel Q will have the form (3):

$-b_i \sin(\omega_0 t) = -3 \sin(\omega_0 t), 3 \sin(\omega_0 t), -\sin(\omega_0 t), \sin(\omega_0 t), 3 \sin(\omega_0 t)$.

For calculations, the also:

- $dT_S=10$ – this is the duration (length) of the continuous signal $a_i(t)$ or $b_i(t)$. It is the duration of the cosine or sine wave assigned ago continuous signal;
- $ilp=100$ – this is the number of points in the period T_S needed to plot the cosine or sine wave (the number of values of these functions in the period T_S);
- n – the number of bits in the $\{q\}$, $n=20$;
- $n1$ – the number of fours of bits in the $\{q\}$, $n1=n/4=5$;
- $tmax$ – this is the simulation time, $tmax=dT_S \cdot n1=10 \cdot 5=50$;
- $nmax$ – this is the number of points chart signal $S(t)$;
- $nmax = ilp \cdot n1=100 \cdot 5=500$;
- $delta$ – this is the distance between two adjacent points on the graph, $delta=dT_S/ilp=10/100=0.1$;
- ω_0 – is the frequency of the cosine and sine wave. ω_0 is chosen so that the total number of times the cosine or sine occurred during the T_S . The result is that in subsequent periods of time T_S , the values of the cosine and sine are repeated, the same. In this example, it is assumed that during the T_S cosine or sine occurs exactly 3 times, $\omega_0=(2 \cdot \pi/dT_S) \cdot 3=1.88495559$. For the purposes of computing, the created table, in which bits of the constellation are assigned the amplitude in the channels of the I and Q.

Table 1. Tabular record of the constellation

The number of the decimal	Bits of the constellation	The amplitude (a_i, b_i)
0	0 0 0 0	(-3, -3)
1	0 0 0 1	(-3, -1)
2	0 0 1 0	(-3, 3)
3	0 0 1 1	(-3, 1)
4	0 1 0 0	(-1, -3)
5	0 1 0 1	(-1, -1)
6	0 1 1 0	(-1, 3)
7	0 1 1 1	(-1, 1)
8	1 0 0 0	(3, -3)
9	1 0 0 1	(3, -1)
10	1 0 1 0	(3, 3)
11	1 0 1 1	(3, 1)
12	1 1 0 0	(1, -3)
13	1 1 0 1	(1, -1)
14	1 1 1 0	(1, 3)
15	1 1 1 1	(1, 1)

On the basis of four bits determine the amplitude of a_i and b_i in the following way: four bits change to a decimal number. The pointer $i=\text{decimal number}+1$ (note: the numbering of the vectors in Matlab starts at 1). Then we read (from table 2) the amplitude of the a_i and b_i . Based on the calculated amplitudes of a_i and b_i , we create constant functions for consecutive fours bits. It is shown in figure 5.

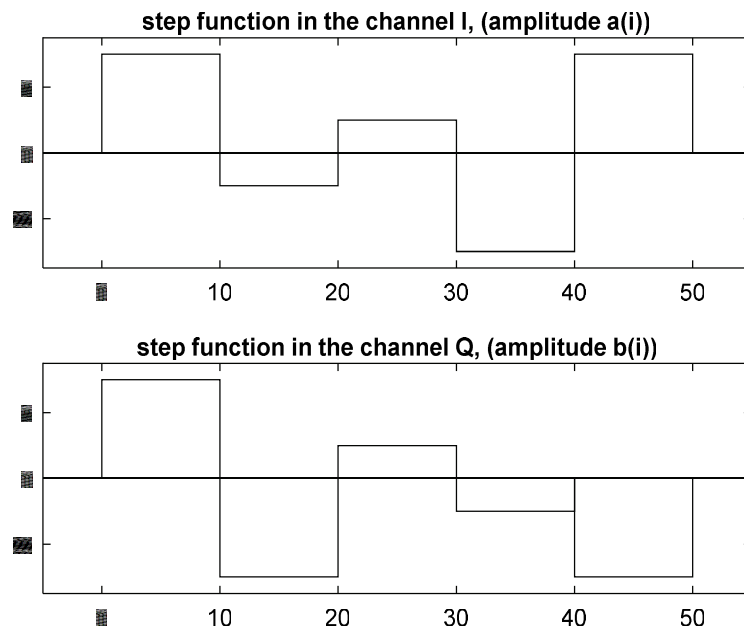


Fig. 5. Output signals from digital to analog converter

Then the first of these signals is multiplied by $\cos(\omega_0 t)$, and the second signal is multiplied by $-\sin(\omega_0 t)$. This is shown in figure 6.

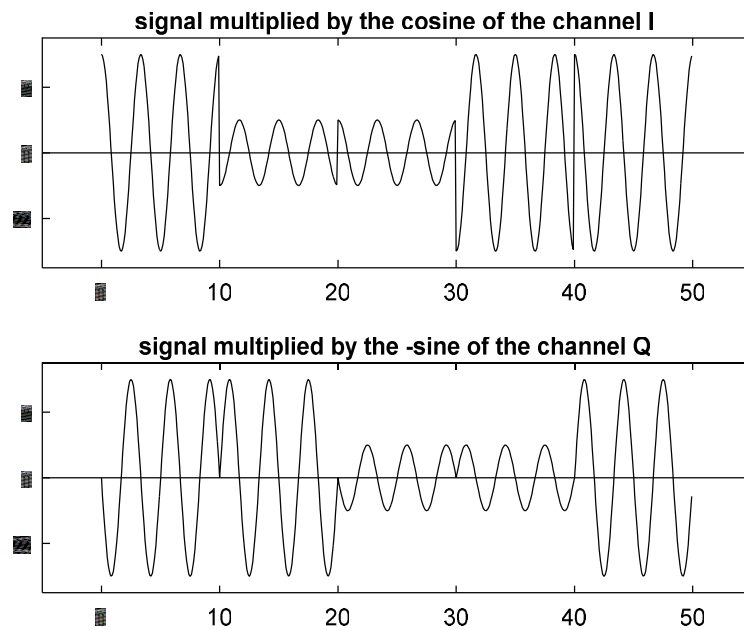


Fig. 6. Carrier signals in the channels I and Q

These two signals are summed to form the resulting signal QAM. It is shown in figure 7.

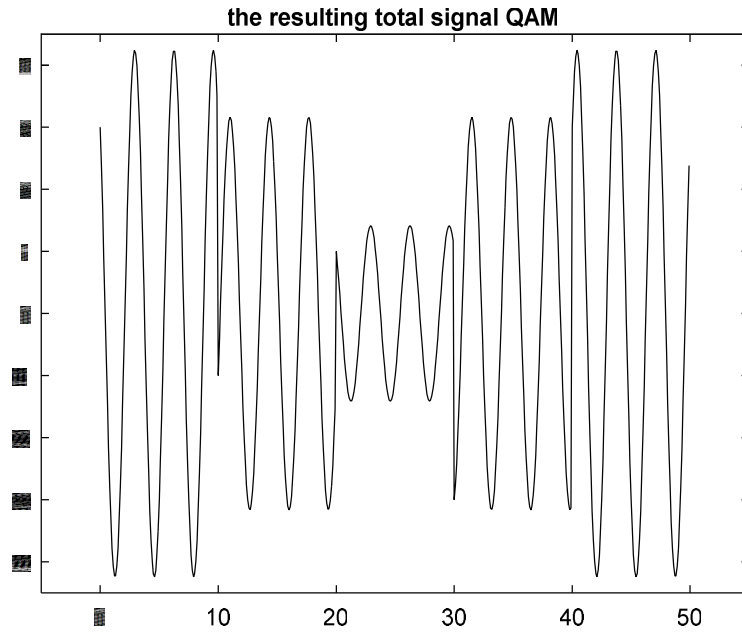


Fig. 7. The resulting signal QAM

It should be noted that in digital simulation, QAM signal M for the example is a vector consisting of 500 values $\{w_j\}$.

4 Simulation of demodulating

Simulation of demodulating QAM signal we will shown in figure 7 and for the constellation in the first figure 3. QAM signal in the digital version is a vector $w = \{w_j\}$, ($j=1, \dots, nmax=500$).

For $S_i(t)$ of the specified formula (3) calculate the values of $S_i(0)$

$$S_i(0) = a_i \cos(\omega_0 \cdot 0) - b_i \sin(\omega_0 \cdot 0) = a_i$$

For each time interval T_S so we have

$$a_i = w_{(i-1)lp+1}$$

To calculate b_i we use equation (3) and the second element the $\{w_j\}$ in each time period T_S . Knowing the previously calculated a_i we get

$$w_{(i-1)lp+2} = a_i \cos(\omega_0 \cdot \delta) - b_i \sin(\omega_0 \cdot \delta)$$

$$b_i = \frac{a_i \cos(\omega_0 \cdot \delta) - w_{(i-1)lp+2}}{\sin(\omega_0 \cdot \delta)}$$

For the data from example 1 formula takes the form

$$b_i = \frac{a_i \cdot 0.98228725 - w_{(i-1)lp+2}}{0.18738131}$$

So based on the first two numbers in the compartment of T_S we get two numbers a_i and b_i . Then based on the constellation and a pair of numbers (a_i, b_i) , you must create a string four bits r_1, r_2, r_3, r_4 . Bits of the constellation of save as decimal numbers. On the basis of the first constellation (figure 3), we can create the array d decimal values for groups of four bits

$$d = \begin{bmatrix} 2 & 6 & 14 & 10 \\ 3 & 7 & 15 & 11 \\ 1 & 5 & 13 & 9 \\ 0 & 4 & 12 & 8 \end{bmatrix}$$

On the basis of a pair of numbers (a_i, b_i) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d .

$$k = -0.5b_i + 2.5$$

$$l = 0.5a_i + 2.5$$

The number of $d(k, l)$ is a decimal number for binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions

$$r = \text{dec2bin}(d(k, l))$$

Then we get a string of four bits $r = \{r_1, r_2, r_3, r_4\}$. Repeat this sequence for each time interval T_S .

Example 2

We trace the demodulation for the QAM obtained in example 1.

For the first time interval T_S ($0 \leq t < 10$) we $w_1=3$, $w_2=2.38471781$. We calculate

$$a_1 = w_1 = 3$$

$$b_1 = \frac{3 \cdot 0.98228725 - 2.38471781}{0.18738131} = \frac{0.56214394}{0.18738131} = 3$$

For a two of numbers (3, 3) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d .

$$k = -0.5b_i + 2.5 = -0.5 \cdot 3 + 2.5 = -1.5 + 2.5 = 1$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot 3 + 2.5 = 1.5 + 2.5 = 4$$

The number of $d(1, 4)=10$ is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions $r = \text{dec2bin}(10)$. We get $r = \{1 \ 0 \ 1 \ 0\}$.

For the second time interval T_S ($10 \leq t < 20$) we $w_{101}=-1$, $w_{102}=-0.42014331$. We calculate

$$a_2 = w_{101} = -1$$

$$b_2 = \frac{-0.98228725 + 0.42014331}{0.18738131} = \frac{-0.56214394}{0.18738131} = -3$$

For a two of numbers (-1, -3) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d .

$$k = -0.5b_i + 2.5 = -0.5 \cdot (-3) + 2.5 = 1.5 + 2.5 = 4$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot (-1) + 2.5 = -0.5 + 2.5 = 2$$

The number of $d(4, 2)=4$ is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions $r = \text{dec2bin}(4)$. We get $r = \{0 \ 1 \ 0 \ 0\}$.

For the third time interval T_S ($20 \leq t < 30$) we $w_{201}=1$, $w_{202}=0.79490594$. We calculate

$$a_3 = w_{201} = 1$$

$$b_3 = \frac{0.98228725 - 0.79490594}{0.18738131} = \frac{0.18738131}{0.18738131} = 1$$

For a two of numbers (1, 1) we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d .

$$k = -0.5b_i + 2.5 = -0.5 \cdot 1 + 2.5 = -0.5 + 2.5 = 2$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot 1 + 2.5 = 0.5 + 2.5 = 3$$

The number of $d(2, 3)=15$ is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions $r=\text{dec2bin}(15)$. We get $r=\{1\ 1\ 1\ 1\}$.

For the fourth time interval T_S ($30 \leq t < 40$) we $w_{301}=-3$, $w_{302}=-2.75948044$. We calculate

$$b_3 = \frac{-3 \cdot 0.98228725 + 2.75948044}{0.18738131} = \frac{a_4 = w_{301} = -3}{0.18738131} + \frac{-2.94686175 + 2.75948044}{0.18738131} + \frac{-0.18738131}{0.18738131} = -1$$

For a two of numbers $(-3, -1)$ we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d .

$$k = -0.5b_i + 2.5 = -0.5 \cdot (-1) + 2.5 = 0.5 + 2.5 = 3$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot (-3) + 2.5 = -1.5 + 2.5 = 1$$

The number of $d(3, 1)=1$ is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions $r=\text{dec2bin}(1)$. We get $r=\{0\ 0\ 0\ 1\}$.

For the fifth time interval T_S ($40 \leq t < 50$) we $w_{401}=3$, $w_{402}=3.50900570$. We calculate

$$b_3 = \frac{3 \cdot 0.98228725 - 3.50900570}{0.18738131} = \frac{a_5 = w_{401} = 3}{0.18738131} + \frac{2.94686175 - 3.50900570}{0.18738131} + \frac{-0.56214395}{0.18738131} = -3$$

For a two of numbers $(3, -3)$ we calculate indicators: k – number of line and l – number of column, for decimal numbers of matrix d .

$$k = -0.5b_i + 2.5 = -0.5 \cdot (-3) + 2.5 = 1.5 + 2.5 = 4$$

$$l = 0.5a_i + 2.5 = 0.5 \cdot 3 + 2.5 = 1.5 + 2.5 = 4$$

The number of $d(4, 4)=8$ is a decimal number to binary string $\{r_1, r_2, r_3, r_4\}$. To get the string we use instructions $r=\text{dec2bin}(8)$. We get $r=\{1\ 0\ 0\ 0\}$.

By connecting together five strings of four bits we get the resulting string decoded $\{1010\ 0100\ 1111\ 0001\ 1000\}$, which is the same as the input string $q=\{q_j\}$.

Conclusion

QAM modulation involves the use of a single frequency bands to transmit two different signals. For a clear separation, in the receiver both signals of information, carrier signals must be shifted against each other in a phase of about 90° [3, 4]. The work illustrates the general principle of QAM. Shows the different diagrams the constellation for 16-QAM modulation. Also made example simulation of digital modulation 16-QAM for 20-bits. Shows also the demodulation method based on the first two QAM signal in each time period T_S . They must, however, be met assumptions about the frequency ω_0 , that early were presented. In this case, the simulation of demodulating QAM signal must be set in the form of a numeric string.



PŘEKRAČUJEME HRANICE
PRZEKACZAMY GRANICE
2014–2020



EVROPSKÁ UNIE / UNIA EUROPEJSKA
EVROPSKÝ FOND PRO REGIONÁLNÍ ROZVOJ
EUROPEJSKI FUNDUSZ ROZWOJU REGIONALNEGO

Projekt jest współfinansowany ze środków Europejskiego Funduszu Rozwoju Regionalnego
oraz z budżetu państwa RP „Przekraczamy Granice”

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SYMULACJA CYFROWA MODULACJI 16-QAM

Streszczenie: W tej pracy przedstawiono zasady modulacji QAM - Quadrature Amplitude Modulation. Dla modulacji 16-QAM przestawiono różne diagramy konstelacji. Pokazano przykład modulacji 16-QAM. Następnie przedstawiono symulację demodulacji. Obliczenia wykonano w Matlabie.

Słowa kluczowe: modulacja, symulacja, diagram konstelacji, kwadraturowa modulacja amplitudowo-fazowa.