The semi-smooth Newton method for Solving the Stokes flow under the leak boundary condition

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Abstract: We consider the Stokes equations under the leak boundary condition. Using the P1-bubble/P1 finite element approximation we get the algebraic optimization problem. Its optimality conditions are the starting point for the algorithm. We use an active set implementation of the semi-smooth Newton method to find the solution. Numerical experiments demonstrate the computational efficiency of an adaptive diagonal preconditioner.

Keywords: Stokes flow, leak boundary condition, semi-smooth Newton method, conjugate gradient method, preconditioning.

1 Introduction

Let Ω be a bounded domain in $\mathbb{R}^2$ with a sufficiently smooth boundary $\partial\Omega$ that is split into three nonempty disjoint parts: $\partial\Omega = \overline{\gamma}_D \cup \overline{\gamma}_N \cup \overline{\gamma}_C$. We consider the model of a viscous incompressible Newtonian fluid modelled by the Stokes system with the Dirichlet and Neumann boundary conditions on γ_D and γ_N, respectively, and with the leak boundary condition of the Navier-Tresca type on γ_C. We are searching for a vector function representing the flow velocity field $\mathbf{u} : \bar{\Omega} \to \mathbb{R}^2$, $\mathbf{u} = (u_1, u_2)$ and a scalar function representing the pressure field $p : \bar{\Omega} \to \mathbb{R}$ so that:

$$\left.\begin{aligned}
-\nu\Delta\mathbf{u} + \nabla p &= \mathbf{f} && \text{in } \Omega,\\
\nabla\cdot\mathbf{u} &= 0 && \text{in } \Omega,\\
\mathbf{u} &= \mathbf{u}_D && \text{on } \gamma_D,\\
\boldsymbol{\sigma} &= \boldsymbol{\sigma}_N && \text{on } \gamma_N,\\
u_t &= 0 && \text{on } \gamma_C,\\
u_n = 0 \Rightarrow |\sigma_n| &\le g && \text{on } \gamma_C,\\
\sigma_n u_n + g|u_n| + \kappa u_n^2 &= 0 && \text{on } \gamma_C,
\end{aligned}\right\} \tag{1}$$

where $\nu > 0$ is the viscosity, $\mathbf{f} : \bar{\Omega} \to \mathbb{R}^2$ describes the forces acting on the fluid, $\mathbf{u}_D : \gamma_D \to \mathbb{R}^2$ and $\boldsymbol{\sigma}_N : \gamma_N \to \mathbb{R}^2$ are the Dirichlet and Neumann boundary data, respectively. Further, $\mathbf{n}$ and

$\mathbf{t}$ are the unit outward normal and tangential vectors on $\partial\Omega$, for which we define the normal and tangential components of the velocity and the stress $u_n = \mathbf{u}\cdot\mathbf{n}$, $u_t = \mathbf{u}\cdot\mathbf{t}$, $\sigma_n = \nu\boldsymbol{\sigma}\cdot\mathbf{n}$, $\sigma_t = \boldsymbol{\sigma}\cdot\mathbf{t}$, respectively, where $\boldsymbol{\sigma} = \nu\partial\mathbf{u}/\partial\mathbf{n} - p\mathbf{n}$ is the stress vector on $\partial\Omega$ in the normal direction corresponding to a non-symmetric tensor. On γ_C we consider the given leak bound $g : \gamma_C \to \mathbb{R}_+$ and the adhesive coefficient $\kappa : \gamma_C \to \mathbb{R}_+$ defining the leak boundary condition. We get the classical Navier law for $g = 0$, while $\kappa = 0$ leads to the Tresca law. We assume that γ_D, γ_N, and γ_C are always non-empty sets.

2 Algebraic formulation

After the mixed finite element approximation based on the P1-bubble/P1 finite elements [8] we arrive at the minimization formulation with the following optimality conditions:

$$\left.\begin{array}{l}
\textit{Find } (\mathbf{u},\mathbf{p},\boldsymbol{s}_n,\boldsymbol{\lambda}_t) \in \mathbb{R}^{n_u}\times\mathbb{R}^{n}\times\mathbb{R}^{n_c}\times\mathbb{R}^{n_c} \textit{ such that}\\
\mathbf{Au} - \mathbf{l} + \mathbf{N}^T\mathbf{s}_n + \mathbf{T}^T\boldsymbol{\lambda}_t + \mathbf{B}^T\mathbf{p} = \mathbf{0},\\
\mathbf{Bu} - \mathbf{Ep} - \mathbf{c} = \mathbf{0},\\
\mathbf{Tu} = \mathbf{0},\\
\left.\begin{array}{l}
(\mathbf{Nu})_i = 0 \Rightarrow |s_{ni}| \le g_i,\\
(\mathbf{Nu})_i > 0 \Rightarrow s_{ni} = g_i + \kappa_i(\mathbf{Nu})_i,\\
(\mathbf{Nu})_i < 0 \Rightarrow s_{ni} = -g_i + \kappa_i(\mathbf{Nu})_i,
\end{array}\right\} \; i \in \mathcal{N},
\end{array}\right\} \tag{2}$$

where $\mathbf{s}_n = \boldsymbol{\lambda}_n + \mathbf{D}(\boldsymbol{\kappa})\mathbf{Nu}$ and $\mathcal{N} = \{1, ..., n_c\}$. Here, $\mathbf{A} \in \mathbb{R}^{n_u\times n_u}$ is the symmetric, positive definite stiffness matrix for the Laplace operator, $\mathbf{l} \in \mathbb{R}^{n_u}$, $\mathbf{B} \in \mathbb{R}^{n\times n_u}$ is the full row rank stiffness matrix for the divergence operator, $\mathbf{T}, \mathbf{N} \in \mathbb{R}^{n_c\times n_u}$ are the full row rank matrices given by the normal and tangential vectors at nodes $\mathbf{x}_i \in \bar{\gamma}_C\backslash\bar{\gamma}_D$, respectively, $\mathbf{D}(\boldsymbol{\kappa}) = \mathrm{diag}(\boldsymbol{\kappa}) \in \mathbb{R}^{n_c\times n_c}$, $\boldsymbol{\kappa} = (\kappa_1, ..., \kappa_{n_c})^T \in \mathbb{R}^{n_c}$, $\kappa_i = h_i\kappa(\mathbf{x}_i)$, $g_i = h_i g(\mathbf{x}_i)$, and h_i is the length of the segment corresponding to $\mathbf{x}_i$, $i \in \mathcal{N}$; n_u is the number of the velocity components, n is the number of the finite elements nodes, and n_c is the number of the leak nodes. The symmetric, positive semidefinite matrix $\mathbf{E} \in \mathbb{R}^{n_c\times n_c}$ and the vector $\mathbf{c} \in \mathbb{R}^{n_c}$ arise from the elimination of the bubble components. While the unknowns $\mathbf{u}$, $\mathbf{p}$ are the vectors of the velocity and pressure components, respectively, $\boldsymbol{\lambda}_t$, $\boldsymbol{\lambda}_n$ are the Lagrange multipliers and $\mathbf{s}_n$ approximates the (negative) shear stress σ_n.

3 Semi-smooth Newton method

Its convenient to use the semi-smooth Newton method to find the solution of (2). Firstly, we reformulate the leak boundary condition in (2) as a nonsmooth equation. We introduce the projection on the interval $[a, b]$,

$$P_{[a,b]}(x) = x - \max\{0, x - b\} + \max\{0, a - x\}, \quad x \in \mathbb{R}$$

and represent the leak boundary condition from (2), see Lemma 1 in Appendix such that

$$\left.\begin{array}{lcll}
(\boldsymbol{N}\mathbf{u})_i & = & \max\{0, \kappa^{-1}(s_{ni} - g_i)\} - \max\{0, -\kappa^{-1}(s_{ni} + g_i)\} & \text{for} \quad \rho_i = \kappa_i > 0,\\
\rho_i(\boldsymbol{N}\mathbf{u})_i & = & \max\{0, s_{ni} - g_i + \rho_i(\boldsymbol{N}\mathbf{u})_i\} - \max\{0, -s_{ni} - g_i - \rho_i(\boldsymbol{N}\mathbf{u})_i\} & \text{for} \quad \kappa_i = 0,
\end{array}\right\}.$$

Firstly, we divide the index set $\mathcal{N}$ into two sets $\mathcal{N}_0$ and $\mathcal{N}_+$ so that $\mathcal{N} = \mathcal{N}_0 \cup \mathcal{N}_+$ as follows:

$$\mathcal{N}_0 = \{i \in \mathcal{N} : \kappa_i = 0\}, \quad \mathcal{N}_+ = \{i \in \mathcal{N} : \kappa_i > 0\}.$$

Let us write the problem (2) as one equation:

$$\boldsymbol{G}(\mathbf{y}) = 0, \tag{3}$$

where $\boldsymbol{G}(\mathbf{y}) = (\boldsymbol{G}_1^\top(\mathbf{y}), \boldsymbol{G}_2^\top(\mathbf{y}), \boldsymbol{G}_3^\top(\mathbf{y}), \boldsymbol{G}_4^\top(\mathbf{y}))^\top$ and $\mathbf{y} = (\mathbf{u}^\top, \mathbf{s}_n^\top, \lambda_t^\top, \boldsymbol{p}^\top)^\top$, where $\boldsymbol{G}_1(\mathbf{y}) = \mathbf{Au} - \mathbf{l} + \mathbf{N}^T\mathbf{s}_n + \mathbf{T}^T\boldsymbol{\lambda}_t + \mathbf{B}^T\mathbf{p}$,

$$\begin{aligned}
\boldsymbol{G}_2(\mathbf{y}) &= \boldsymbol{N}_+\mathbf{u} - \max\{0, \boldsymbol{D}_{\kappa_+}^{-1}(s_{n_+} - g_+)\} + \max\{0, -\boldsymbol{D}_{\kappa_+}^{-1}(s_{n_+} + g_+)\}, \\
\boldsymbol{G}_3(\mathbf{y}) &= \rho\boldsymbol{N}_0\mathbf{u} - \max\{0, s_{n_0} - g_0 + \rho\boldsymbol{N}_0\mathbf{u}\} + \max\{0, -s_{n_0} - g_0 - \rho\boldsymbol{N}_0\mathbf{u}\},
\end{aligned}$$

$\boldsymbol{G}_4(\mathbf{y}) = \mathbf{Tu}$ and $\boldsymbol{G}_5(\mathbf{y}) = \mathbf{Bu} - \mathbf{Ep} - \mathbf{c} = \mathbf{0}$.

The equation (3) can be solved by the semi-smooth Newton method, because $\boldsymbol{G}$ is semi-mooth in the sense of [5].

4 Algorithm

According to the division of $\mathcal{N}$, we define for each $\mathbf{y}^k$ two types of the inactive sets

$$\mathcal{I}_+^+ = \{i \in \mathcal{N}_+ : s_{ni}^k \geq g_i\}, \quad \mathcal{I}_+^- = \{i \in \mathcal{N}_+ : s_{ni}^k \leq -g_i\}$$

and

$$\mathcal{I}_0^+ = \{i \in \mathcal{N}_0 : s_{ni}^k + \rho(Nu)_i \geq g_i\}, \quad \mathcal{I}_0^- = \{i \in \mathcal{N}_0 : s_{ni}^k + \rho(Nu)_i \leq -g_i\}$$

or

$$\mathcal{I}_0^+ = \{i \in \mathcal{N}_0 : s_{ni}^k - \rho r_i \geq g_i\}, \quad \mathcal{I}_0^- = \{i \in \mathcal{N}_0 : s_{ni}^k - \rho r_i \leq -g_i\}$$

and the active sets as their complements $\mathcal{A}_+ = \mathcal{N}_+ \setminus (\mathcal{I}_+^+ \cup \mathcal{I}_+^-)$ and $\mathcal{A}_0 = \mathcal{N}_0 \setminus (\mathcal{I}_0^+ \cup \mathcal{I}_0^-)$.

Further, we define the indicator matrices $\mathbf{D}(\mathcal{I}_+^-)$, $\mathbf{D}(\mathcal{I}_+^+)$ and $\mathbf{D}(\mathcal{I}_0^-)$, $\mathbf{D}(\mathcal{I}_0^+)$, respectively. Note that the indicator matrix to $\mathscr{S}_+ \subseteq \mathcal{N}_+$ is given by $\mathbf{D}(\mathscr{S}_+) = \operatorname{diag}(s_1, ..., s_{n_{c+}}) \in \mathbb{R}^{n_{c+} \times n_{c+}}$, where $n_{c+} := |\mathscr{S}_+| \leq n_c$, with $s_i = 1$ for $i \in \mathscr{S}_+$ and $s_i = 0$ if $i \notin \mathscr{S}_+$. The new iterate $\mathbf{y}^{k+1}$ is computed by solving the following linear system. Moreover, in the case $\mathcal{N}_0 \neq \emptyset$ we get $\mathbf{s}_{n,\mathcal{A}_0} = \boldsymbol{\lambda}_{n,\mathcal{A}_0}$ and set:

$$\mathbf{s}_{n,\mathcal{I}_0^+} = \mathbf{g}_{\mathcal{I}_0^+}, \qquad \mathbf{s}_{n,\mathcal{I}_0^-} = -\mathbf{g}_{\mathcal{I}_0^-}.$$

$$\left(\begin{array}{c|cccc}
\mathbf{A} & \mathbf{N}_+^T & \mathbf{N}_{\mathcal{A}_0}^T & \mathbf{T}^T & \mathbf{B}^T \\
\hline
\mathbf{N}_+ & -\mathbf{D}(\boldsymbol{\kappa}_+)^{-1}\mathbf{D}(\mathcal{I}_+^+ \cup \mathcal{I}_+^-) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{N}_{\mathcal{A}_0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{T} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{E}
\end{array}\right)
\begin{pmatrix}
\mathbf{u}^{k+1} \\ \mathbf{s}_{n+}^{k+1} \\ \boldsymbol{\lambda}_{n\mathcal{A}_0}^{k+1} \\ \boldsymbol{\lambda}_t^{k+1} \\ \mathbf{p}^{k+1}
\end{pmatrix}$$

$$= \begin{pmatrix}
\mathbf{l} - \mathbf{N}_{\mathcal{I}_0^+}^T\mathbf{g}_{\mathcal{I}_0^+} + \mathbf{N}_{\mathcal{I}_0^-}^T\mathbf{g}_{\mathcal{I}_0^-} \\
\mathbf{D}(\boldsymbol{\kappa}_+)^{-1}(\mathbf{D}(\mathcal{I}_+^-) - \mathbf{D}(\mathcal{I}_+^+))\mathbf{g}_+ \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{c}
\end{pmatrix}. \tag{4}$$

The conjugate gradient method with adaptive precision control is used to find the solution. As you can see in the section Numerical experiments, it is convenient to use a preconditioner.

5 Preconditioning

To solve bigger linear systems and more complex meshes we use the Schur complement to solve (4):

$$\boldsymbol{S}^k \mathbf{r}^{k+1} = \boldsymbol{C}\boldsymbol{A}^{-1}\mathbf{l} - \mathbf{h}^k,$$

where $\boldsymbol{S}^k = \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{C}^\top + \bar{\boldsymbol{E}}^k$, where

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{N}_+ \\ \boldsymbol{N}_{\mathcal{A}_0} \\ \boldsymbol{T} \\ \boldsymbol{B} \end{pmatrix}, \quad \bar{\boldsymbol{E}}^k \begin{pmatrix} \mathbf{D}(\boldsymbol{\kappa}_+)^{-1}\mathbf{D}(\mathcal{I}_+^+ \cup \mathcal{I}_+^-) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{E} \end{pmatrix}$$

and

$$\mathbf{r}^{k+1} = \begin{pmatrix} \mathbf{s}_{n_+}^{k+1} \\ \lambda_{n\mathcal{A}_n}^{k+1} \\ \lambda_t^{k+1} \\ \boldsymbol{p}^{k+1} \end{pmatrix}, \mathbf{h}^k = \begin{pmatrix} \mathbf{D}(\boldsymbol{\kappa}_+)^{-1}(\mathbf{D}(\mathcal{I}_+^-) - \mathbf{D}(\mathcal{I}_+^+))\mathbf{g}_+ \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c} \end{pmatrix}.$$

We use the diagonal preconditioner

$$\boldsymbol{P}^k = diag\boldsymbol{S}^k.$$

6 Numerical experiments

We consider the L-shaped domain $\Omega = (0,5) \times (0,2) \setminus \overline{\mathcal{S}}$, $\mathcal{S} = (0,1) \times (0,1)$ with $\nu = 1$ and $\mathbf{f} = \mathbf{0}$; $\gamma_D = \gamma_{top} \cup \gamma_{left}$ with $\gamma_{top} = (0,5) \times \{2\}$, $\gamma_{left} = \{0\} \times (1,2)$, $\mathbf{u}_{D|\gamma_{top}} = \mathbf{0}$, and $\mathbf{u}_{D|\gamma_{left}} = (4(x_2 - 2)(1 - x_2), 0)$; $\gamma_N = \{5\} \times (0,2)$ with $\boldsymbol{\sigma}_N = \mathbf{0}$; $\gamma_C = \partial\Omega \setminus \overline{(\gamma_D \cup \gamma_N)}$ with $g = 10$ for $x < 1$ and else $g = 1$. In tables below we report $iter/n_S$, where $iter$ is the number of the outer (Newton) iterations, while n_S denotes the total number of the matrix-vector multiplications by the Schur complements. Note that n_S determines the computational efficiency. The computational efficiency without preconditioning for different adhesive coefficients κ is shown in Table 1. One can see that n_S increases considerable for finer meshes and smaller κ. This unacceptable effect is eliminated by preconditioning, as it is seen from Table 2.

Table 1: The computational complexity for different κ without preconditioning.

$n_u/n/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$	$\kappa = 0$
344/206/32	15/1358	15/1139	20/1614	23/1984	23/1919	10/865
1352/744/64	18/3143	18/2666	19/2888	19/3817	24/4925	15/2047
5366/2819/128	16/4248	18/5440	23/7591	20/8558	23/12066	18/3889
21386/10965/256	17/6526	21/11170	17/14308	17/14207	26/27023	17/5468
85394/43241/512	21/17776	22/20523	24/24900	21/29581	24/34877	18/8640

Table 2: The computational complexity for different κ with preconditioning.

$n_u/n/n_c$	$\kappa = 1$	$\kappa = 0.5$	$\kappa = 0.1$	$\kappa = 0.01$	$\kappa = 0.001$	$\kappa = 0$
344/206/32	8/162	10/179	10/163	16/186	7/185	7/152
1352/744/64	9/191	9/180	10/179	10/140	6/82	9/205
5366/2819/128	9/226	10/218	13/234	19/303	23/294	9/233
21386/10965/256	8/256	10/291	12/250	17/351	24/435	7/213
85394/43241/512	11/407	10/331	14/352	8/166	9/177	10/388

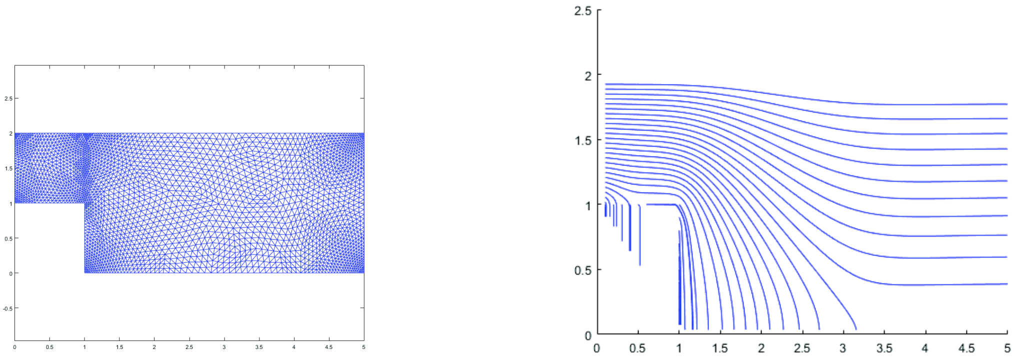

Figure 1: Finite element approximation and velocity field

Appendix

Lemma 1 *Let* $\lambda, u \in \mathbb{R}^1$, $g \geq 0$, $\kappa \geq 0$. *The relations*

$$\left.\begin{array}{rcrcl} |\lambda| & \leq & g & \Rightarrow & u = 0 \\ \lambda & > & g & \Rightarrow & \lambda = g + \kappa u \\ \lambda & < & -g & \Rightarrow & \lambda = -g + \kappa u \end{array}\right\} \tag{5}$$

hold iff

$$\psi(\lambda, u) = 0,$$

where $\psi(\lambda, u) := \rho u - \max\{0, \lambda - g + (\rho - \kappa)u\} + \max\{0, -\lambda - g + (\kappa - \rho)u\}$, *see Figure 2.*

Proof: We assume $g > 0$, as $g = 0$ is trivial. First we prove the implication '$\Rightarrow$'. The relations (5) are satisfied. In the first case $|\lambda| \leq g$ and $u = 0$ we get $\psi(\lambda, 0) = 0 - \max\{0, \lambda - g\} + \max\{0, -\lambda - g\} = 0$. In the second case $\lambda > g$ and $\lambda = g + \kappa u$ we get $\psi(\lambda, u) := \rho u - \max\{0, \rho u\} + \max\{0, -2g - \rho u\} = \rho u - \rho u + 0 = 0$. In the third case $\lambda < -g$ and $\lambda = -g + \kappa u$ we get $\psi(\lambda, u) := \rho u - \max\{0, -2g + \rho u\} + \max\{0, -\rho u\} = \rho u - 0 - \rho u = 0$. To prove the opposite implication, we start from $\rho u = \max\{0, \lambda - g + (\rho - \kappa)u\} - \max\{0, -\lambda - g + (\kappa - \rho)u\}$. If $|\lambda| \leq g$ then $\lambda - g + (\rho - \kappa)u > 0$ and $-\lambda - g + (\kappa - \rho)u < 0$ or $-\lambda - g + (\kappa - \rho)u > 0$ and $\lambda - g + (\rho - \kappa)u < 0$. In the first case we suppose $\lambda - g + (\rho - \kappa)u = \rho u$ and $-\lambda - g + (\kappa - \rho)u < 0$, in the second case $-\lambda - g + (\kappa - \rho)u = -\rho u$ and $\lambda - g + (\rho - \kappa)u < 0$, $\lambda - g \leq 0$ as well as $-\lambda - g \leq 0$, then $u = 0$. Analogously, in the case $\lambda > g$ we get $\lambda = g + \kappa u$ and $\lambda = -g + \kappa u$ the last case. $\square$

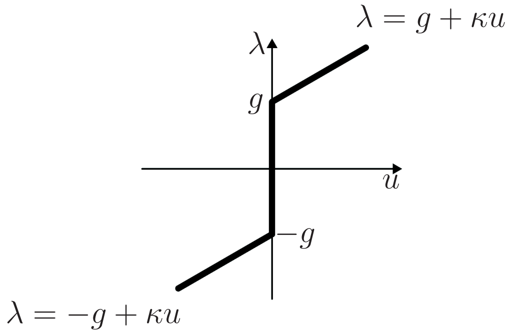

Figure 2: Projection

Conclusion

We have analysed the numerical solution of the Stokes flow with the leak boundary condition of friction type. The analysis is analogical to the analysis of the Stokes flow with stick-slip boundary condition. The numerical experiments have shown that it is necessary to use the preconditioner in the conjugate gradient method to implement a computationaly efficient solver.

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Nehladká Newtonova metoda pro řešení Stokesova proudění s průsakem

Abstrakt (Streszczenie): Uvažujeme Stokesovu úlohu s průsakem. Po aproximaci pomocí konečných prvků P1-bubble/P1 dostáváme algebraický optimalizační problém. Jeho podmínky optimality jsou výchozím bodem pro algoritmus. Pro nalezení řešení problému použijeme nehladkou Newtonovu metodu implementovanou pomocí aktivních množin. Numerické experimenty demonstrují výpočetní efektivitu adaptivního diagonálního předpodmíňovače.

Klíčová slova (Słowa kluczowe): Stokesovo proudění, nehladká Newtonova metoda, průsak, metoda konjugovanych gradientu, předpodmínění.