Hamiltonian system in dimension 4

SMETANOVÁ Dana

Institute of Technology and Business in České Budějovice, Okružní 317/10, 370 01 České Budějovice, CZ
E-mail: smetanova@mail.vste.cz

Abstract: The aim of this paper is announce some recent results concerning the Hamiltonian theory. The case of first order Hamiltonian systems related to an affine second order Euler–Lagrange form is studied. In dimension 4 the structure of Hamiltonian systems (i.e., Lepagean equivalent of an Euler–Lagrange form) is found.

Keywords: Euler–Lagrange form, Lepagean equivalent of Euler–Lagrange form, Hamiltonian system.

1 Introduction

Throughout the paper all manifolds and mappings are smooth and summation convention is used. We consider a fibered manifold (i.e., surjective submersion) $\pi : Y \to X$, dim $X = n$, dim $Y = n + m$, its r-jet prolongation $\pi_r : J^rY \to X$, $r \geq 1$ and canonical jet projections $\pi_{r,k} : J^rY \to J^kY$, $0 \leq k \leq r$ (with an obvious notations $J^0Y = Y$). A fibered chart on Y (resp. associated fibered chart on J^rY) is denoted by (V, ψ), $\psi = (x^i, y^\sigma)$ (resp. (V_r, ψ_r), $\psi_r = (x^i, y^\sigma, y^\sigma_i, \ldots, y^\sigma_{i_1 \ldots i_r})$).

A vector field ξ on J^rY is called π_r-*vertical* if it projects onto the zero vector field on X. A q-form η on J^rY is called π_r-*horizontal* if $i_\xi \eta = 0$ for every π_r-vertical vector field ξ on J^rY.

The fibered structure of Y induces a morphism h, of exterior algebras, defined by the condition $J^r\gamma^*\eta = J^{r+1}\gamma^* h\eta$ for every section γ of π, and called *horizontalization*. Apparently, horizontalization is an $\mathbb{R}$-linear wedge product preserving mapping such that applied to a function f and to the elements of the canonical basis of 1-forms $(dx^i, dy^\sigma, dy^\sigma_i, \ldots, dy^\sigma_{i_1 \ldots i_r})$ on J^rY gives

$$hf = f \circ \pi_{r+1,r},\ hdx^i = dx^i,\ hdy^\sigma = y^\sigma_l dx^l, \ldots, hdy^\sigma_{i_1 \ldots i_r} = y^\sigma_{i_1 \ldots i_r l} dx^l.$$

A q-form η on J^rY is called *contact* if $h\eta = 0$. A contact q-form η on J^rY is called 1-*contact* if for every π_r-vertical vector field ξ on J^rY the $(q-1)$-form $i_\xi \eta$ is horizontal. A contact q-form η on J^rY is called i-*contact* if for every π_r-vertical vector field ξ on J^rY the $(q-1)$-form $i_\xi \eta$ is (i-1)-contact.

Recall that every q-form η on J^rY admits a unique (canonical) decomposition into a sum of q-forms on $J^{r+1}Y$ as follows:

$$\pi^*_{r+1,r}\eta = h\eta + \sum_{k=1}^{q} p_k\eta,$$

where $h\eta$ is a horizontal form, called the *horizontal part of* η, and $p_k\eta$, $1 \leq k \leq q$, is a *k-contact part of* η (see [3]).

We use the following notations:

$$\omega_0 = dx^1 \wedge dx^2 \wedge \cdots \wedge dx^n,\ \omega_i = i_{\partial/\partial x^i}\omega_0,\ \omega_{ij} = i_{\partial/\partial x^j}\omega_i,$$

and

$$\omega^\sigma = dy^\sigma - y^\sigma_j dx^j,\ \ldots\ ,\ \omega^\sigma_{i_1 i_2 \ldots i_k} = dy^\sigma_{i_1 i_2 \ldots i_k} - y^\sigma_{i_1 i_2 \ldots i_k j} dx^j$$

For more details on fibered manifolds and the corresponding geometric stuctures we refer e.g. to [6].

In this section we briefly recall basic concepts on Lepagean equivalents of Euler–Lagrange forms and generalized Hamiltonian field theory, due to Krupková [4].

By an *r-th order Lagrangian* we shal mean a horizontal n-form λ on J^rY.

A *closed* $(n+1)$-form α is called a *Lepagean equivalent of an Euler–Lagrange form* $E = E_\sigma \omega^\sigma \wedge \omega_0$ if $p_1\alpha = E$.

Recall that the Euler–Lagrange form corresponding to an r-th order $\lambda = L\omega_0$ is the following $(n+1)$-form of order $\leq 2r$

$$E = \left(\frac{\partial L}{\partial y^\sigma} + \sum_{l=1}^{r} (-1)^l d_{p_1} d_{p_2} \ldots d_{p_l} \frac{\partial L}{\partial y^\sigma_{p_1 \ldots p_l}} \right) \omega^\sigma \wedge \omega_0.$$

The family of Lepagean equivalents of E is also called a *Lagrangian system*, and denoted by $[\alpha]$. A (single) Lepagean equivalent α of E on J^sY is also called a *Hamiltonian system of order s.*

2 Hamiltonian Systems.

We shall consider dim $X = 4$ and a second order Euler–Lagrange form $E = E_\nu\ \omega^\nu \wedge \omega_0$ which coefficients E_ν are affine in the second derivatives, i.e.,

$$E_\nu = A_\nu + B^{kl}_{\nu\sigma} y^\sigma_{kl}, \tag{1}$$

where A_ν and $B^{kl}_{\nu\sigma}$ do not depend on second derivatives.

In what follows, we shall study first order Hamiltonian systems (i.e., $s = 1$) corresponding to a Lepagean equivalents of such Euler–Lagrange form. In dimension 4 the 1st order Hamiltonian systems admit a decomposition

$$\pi^*_{2,1}\alpha = p_1\alpha + p_2\alpha + p_3\alpha + p_4\alpha + p_5\alpha,$$

Keeping notations introduced so far, we write

$$\begin{aligned}
\alpha &= E_\sigma\, \omega^\sigma \wedge \omega_0 + F^i_{\sigma\nu}\, \omega^\sigma \wedge \omega^\nu \wedge \omega_i + F^{ij}_{\sigma\nu}\, \omega^\sigma \wedge \omega^\nu_i \wedge \omega_j \\
&+ F^{ijk}_{\sigma\nu}\, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega_k + G^{ij}_{\sigma\nu\kappa}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_{ij} \\
&+ G^{ijk}_{\sigma\nu\kappa}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega_{jk} + G^{ijkl}_{\sigma\nu\kappa}\, \omega^\sigma \wedge \omega^\nu_i \wedge \omega^\kappa_j \wedge \omega_{kl} \\
&+ G^{ijklm}_{\sigma\nu\kappa}\, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega^\kappa_k \wedge \omega_{lm} + K^{ijk}_{\sigma\nu\kappa\beta}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_{ijk} \\
&+ K^{ijkl}_{\sigma\nu\kappa\beta}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega_{jkl} \\
&+ K^{ijklm}_{\sigma\nu\kappa\beta}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega^\beta_j \wedge \omega_{jkl} \\
&+ K^{ijklmn}_{\sigma\nu\kappa\beta}\, \omega^\sigma \wedge \omega^\nu_i \wedge \omega^\kappa_j \wedge \omega^\beta_k \wedge \omega_{lmn} \\
&+ K^{ijklmno}_{\sigma\nu\kappa\beta}\, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega^\kappa_k \wedge \omega^\beta_l \wedge \omega_{mno} \\
&+ M^{ijkl}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma \wedge \omega_{ijkl} \\
&+ M^{ijklm}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma_i \wedge \omega_{jklm} \\
&+ M^{ijklmn}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega^\gamma_j \wedge \omega_{klmn} \\
&+ M^{ijklmno}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega^\beta_j \wedge \omega^\gamma_k \wedge \omega_{lmno} \\
&+ M^{ijklmnop}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu_i \wedge \omega^\kappa_j \wedge \omega^\beta_k \wedge \omega^\gamma_l \wedge \omega_{mnop} \\
&+ M^{ijklmnopq}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma_i \wedge \omega^\nu_j \wedge \omega^\kappa_k \wedge \omega^\beta_l \wedge \omega^\gamma_m \wedge \omega_{nopq},
\end{aligned} \tag{2}$$

Lemma 2.1. *Let* dim $X = 4$. *Let $E = E_\nu \omega^\nu \wedge \omega_0$ be a second order Euler–Lagrange form with coefficients E_ν satisfying (1), and let α be a Hamiltonian system of the form (2). Then the functions $F^{ijk}_{\sigma\nu}$, $G^{ijkl}_{\sigma\nu}$, $G^{ijklm}_{\sigma\nu}$, $K^{ijklm}_{\sigma\nu}$, $K^{ijklmn}_{\sigma\nu}$, $K^{ijklmno}_{\sigma\nu}$, $M^{ijklmn}_{\sigma\nu}$, $M^{ijklmno}_{\sigma\nu}$, $M^{ijklmnop}_{\sigma\nu}$, $M^{ijklmnopq}_{\sigma\nu}$ do not depend on E_σ.*

Proof. Proof of the lemma follows from the explicit computation of $d\alpha = 0$. □

One can see from the above lemma that the functions $F^{ijk}_{\sigma\nu}$, $G^{ijkl}_{\sigma\nu}$, $G^{ijklm}_{\sigma\nu}$, $K^{ijklm}_{\sigma\nu}$, $K^{ijklmn}_{\sigma\nu}$, $K^{ijklmno}_{\sigma\nu}$, $M^{ijklmn}_{\sigma\nu}$, $M^{ijklmno}_{\sigma\nu}$, $M^{ijklmnop}_{\sigma\nu}$, $M^{ijklmnopq}_{\sigma\nu}$ do not depend on E_σ (cf. [4]). The invariant choice is

$$\begin{aligned}
&F^{ijk}_{\sigma\nu} = G^{ijkl}_{\sigma\nu} = G^{ijklm}_{\sigma\nu} = K^{ijklm}_{\sigma\nu} = K^{ijklmn}_{\sigma\nu} = K^{ijklmno}_{\sigma\nu} = 0, \\
&M^{ijklmn}_{\sigma\nu} = M^{ijklmno}_{\sigma\nu} = M^{ijklmnop}_{\sigma\nu} = M^{ijklmnopq}_{\sigma\nu} = 0,
\end{aligned}$$

and we obtain new Hamiltonian system of the form

$$\begin{aligned}
\bar{\alpha} &= E_\sigma\, \omega^\sigma \wedge \omega_0 + F^i_{\sigma\nu}\, \omega^\sigma \wedge \omega^\nu \wedge \omega_i + F^{ij}_{\sigma\nu}\, \omega^\sigma \wedge \omega^\nu_i \wedge \omega_j \\
&+ G^{ij}_{\sigma\nu\kappa}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_{ij} + G^{ijk}_{\sigma\nu\kappa}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa_i \wedge \omega_{jk} \\
&+ K^{ijk}_{\sigma\nu\kappa\beta}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_{ijk} + K^{ijkl}_{\sigma\nu\kappa\beta}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta_i \wedge \omega_{jkl} \\
&+ M^{ijkl}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma \wedge \omega_{ijkl} \\
&+ M^{ijklm}_{\sigma\nu\kappa\beta\gamma}\, \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma_i \wedge \omega_{jklm},
\end{aligned} \tag{3}$$

where $F^i_{\sigma\nu}$ are skew-symmetric in the indices $\sigma\nu$, $G^{ij}_{\sigma\nu\kappa}$ are skew-symmetric in the indices $\sigma\nu\kappa$ and skew-symmetric in the ij, $G^{ijk}_{\sigma\nu\kappa}$ are skew-symmetric in the indices $\sigma\nu$ and skew-symmetric in the jk, $K^{ijk}_{\sigma\nu\kappa\beta}$ are skew-symmetric in the indices $\sigma\nu\kappa\beta$ and skew-symmetric in the ijk,

$K^{ijkl}_{\sigma\nu\kappa\beta}$ are skew-symmetric in the indices $\sigma\nu\kappa$ and skew-symmetric in the jkl, $M^{ijkl}_{\sigma\nu\kappa\beta\gamma}$ are skew-symmetric in the indices $\sigma\nu\kappa\beta\gamma$ and skew-symmetric in the $ijkl$, $M^{ijklm}_{\sigma\nu\kappa\beta\gamma}$ are skew-symmetric in the indices $\sigma\nu\kappa\beta$ and skew-symmetric in the $jklm$.

Now the functions on the Hamiltonian system depend on coefficients of Euler–Lagrange form. In the following theorem the stucture of the functions on the Hamiltonian system $\bar{\alpha}$ (3) is studied.

Theorem 2.2. *Let* $\dim X = 4$. *Let* $E = E_\nu \omega^\nu \wedge \omega_0$ *be a second order Euler–Lagrange form with coefficients* E_ν *satisfying (1), and let* $\bar{\alpha}$ *be a Hamiltonian system of the form (3). Then the following coeffient conditions are satisfied*

1) $F^{ij}_{\sigma\nu} = \frac{\partial E_\sigma}{\partial y^\nu_{ij}} + f^{ij}_{\sigma\nu}$, *where* $f^{ij}_{\sigma\nu}$ *are arbitrary functions satisfying* $(f^{ij}_{\sigma\nu})_{\mathrm{sym}(ij)} = 0$ *and* $f^{ij}_{\sigma\nu}$ *do not depend on second derivatives.*

2) $F^{i}_{\sigma\nu} = -\frac{1}{2}\left(\frac{\partial E_\nu}{\partial y^\sigma_i} - d_j F^{ij}_{\nu\sigma}\right)_{\mathrm{alt}(\sigma\nu)}$.

3) $G^{ij}_{\sigma\nu\kappa} = \frac{1}{6}\left(\frac{\partial F^{j}_{\nu\kappa}}{\partial y^\sigma_i} - \frac{\partial F^{ij}_{\nu\sigma}}{\partial y^\kappa}\right)_{\mathrm{alt}(\nu\kappa)} + g^{ij}_{\sigma\nu\kappa}$, *where* $(g^{ij}_{\sigma\nu\kappa})_{\mathrm{alt}(\nu\kappa)} = 0$ *and* $g^{ij}_{\sigma\nu\kappa}$ *do not depend on second derivatives.*

4) $G^{ijk}_{\sigma\nu\kappa} = \left(\frac{\partial F^{ij}_{\nu\kappa}}{\partial y^\sigma_k}\right)_{\mathrm{alt}((\sigma k)(\kappa i))} + g^{ijk}_{\sigma\nu\kappa}$, *where* $(g^{ijk}_{\sigma\nu\kappa})_{\mathrm{alt}((\sigma k)(\kappa i))} = 0$ *and* $g^{ijk}_{\sigma\nu\kappa}$ *do not depend on second derivatives.*

5) $K^{ijq}_{\sigma\nu\kappa\beta} = -\frac{1}{12}\left(\frac{\partial G^{ij}_{\nu\kappa\beta}}{\partial y^\sigma_q} - \frac{\partial G^{qij}_{\nu\kappa\sigma}}{\partial y^\beta}\right)_{\mathrm{alt}(\nu\kappa\beta)} + k^{ijq}_{\sigma\nu\kappa\beta}$, *where* $(k^{ijq}_{\sigma\nu\kappa\beta})_{\mathrm{alt}(\nu\kappa\beta)} = 0$ *and* $k^{ijq}_{\sigma\nu\kappa\beta}$ *do not depend on second derivatives.*

6) $K^{ijql}_{\sigma\nu\kappa\beta} = -\frac{1}{9}\left(\frac{\partial G^{ijq}_{\nu\kappa\beta}}{\partial y^\sigma_l}\right)_{\mathrm{alt}(\nu\kappa),\ \mathrm{alt}((\sigma l)(\beta i))} + k^{ijql}_{\sigma\nu\kappa\beta}$, *where the functions* $(k^{ijq}_{\sigma\nu\kappa\beta})_{\mathrm{alt}(\nu\kappa),\ \mathrm{alt}((\sigma l)(\beta i))} = 0$ *and* $k^{ijq}_{\sigma\nu\kappa\beta}$ *do not depend on second derivatives.*

7) $M^{ijkl}_{\sigma\nu\kappa\beta\gamma} = \frac{1}{20}\left(\frac{\partial K^{ijk}_{\kappa\beta\gamma\nu}}{\partial y^\sigma_l} - \frac{\partial K^{lijk}_{\sigma\kappa\beta\gamma}}{\partial y^\nu}\right)_{\mathrm{alt}(\nu\kappa\beta\gamma))} + m^{ijql}_{\sigma\nu\kappa\beta\gamma}$, *where the functions* $(m^{ijq}_{\sigma\nu\kappa\beta\gamma})_{\mathrm{alt}(\nu\kappa\beta\gamma))} = 0$ *and* $m^{ijq}_{\sigma\nu\kappa\beta}$ *do not depend on second derivatives.*

8) $M^{ijklq}_{\sigma\nu\kappa\beta\gamma} = \frac{1}{16}\left(\frac{\partial K^{ijkl}_{\nu\kappa\beta\gamma}}{\partial y^\sigma_q}\right)_{\mathrm{alt}(\nu\kappa\beta),\ \mathrm{alt}((\sigma q)(\gamma i))} + m^{ijklq}_{\sigma\nu\kappa\beta\gamma}$, *where functions* $(k^{ijq}_{\sigma\nu\kappa\beta\gamma})_{\mathrm{alt}(\nu\kappa\beta),\ \mathrm{alt}((\sigma q)(\gamma i))} = 0$ *and* $m^{ijklq}_{\sigma\nu\kappa\beta\gamma}$ *do not depend on second derivatives.*

Where sym() *means the symmetrization in the indicated multiindices and* alt() *means skew-symmetrization in the indicated multiindices.*

Proof. Proof of the theorem follows from facts that the Hamiltonian systems is of first order and from the explicit computation of $d\bar{\alpha} = 0$. □

The Hamiltonian system (3) $\bar{\alpha}$ admits noninvariant decomposition

$$\bar{\alpha} = \alpha_E + \phi \tag{4}$$

where ϕ does not depend on the Euler–Lagrange form and

$$
\begin{aligned}
\alpha_E &= E_\sigma\, \omega^\sigma \wedge \omega_0 - \frac{1}{2}\left(\frac{\partial E_\nu}{\partial y_i^\sigma} - d_j \frac{\partial E_\nu}{\partial y_{ij}^\sigma}\right) \omega^\sigma \wedge \omega^\nu \wedge \omega_i \qquad (5)\\
&+ \frac{\partial E_\sigma}{\partial y_{ij}^\nu}\, \omega^\sigma \wedge \omega_i^\nu \wedge \omega_j\\
&- \frac{1}{12}\left(\frac{\partial^2 E_\kappa}{\partial y_i^\sigma \partial y_j^\nu} - d_k \frac{\partial^2 E_\kappa}{\partial y_i^\sigma \partial y_{jk}^\nu}\right) \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_{ij}\\
&+ \left(\frac{\partial^2 E_\nu}{\partial y_k^\sigma \partial y_{ij}^\kappa}\right)_{\mathrm{alt}((\sigma k)(\kappa i))} \omega^\sigma \wedge \omega^\nu \wedge \omega_i^\kappa \wedge \omega_{jk}\\
&+ \frac{1}{144}\left(\frac{\partial^3 E_\beta}{\partial y_k^\sigma \partial y_i^\nu \partial y_j^\kappa} - d_l \frac{\partial^3 E_\beta}{\partial y_k^\sigma \partial y_i^\nu \partial y_{jl}^\kappa}\right) \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_{ijk}\\
&- \frac{1}{9}\left(\frac{\partial^3 E_\kappa}{\partial y_l^\sigma \partial y_k^\nu \partial y_{ij}^\beta}\right)_{\mathrm{alt}((\sigma l)(\beta i))} \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega_i^\beta \wedge \omega_{jkl}\\
&+ \frac{1}{2880}\left(\frac{\partial^4 E_\nu}{\partial y_l^\sigma \partial y_k^\kappa \partial y_i^\beta \partial y_j^\gamma} - d_p \frac{\partial^4 E_\nu}{\partial y_l^\sigma \partial y_k^\kappa \partial y_i^\beta \partial y_{jp}^\gamma}\right)\\
&\quad \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega^\gamma \wedge \omega_{ijkl}\\
&- \frac{1}{144}\left(\frac{\partial^4 E_\kappa}{\partial y_m^\sigma \partial y_l^\nu \partial y_k^\beta \partial y_{ij}^\gamma}\right)_{\mathrm{alt}((\sigma m)(\gamma i))} \omega^\sigma \wedge \omega^\nu \wedge \omega^\kappa \wedge \omega^\beta \wedge \omega_i^\gamma \wedge \omega_{jklm},
\end{aligned}
$$

depends on derivatives of coefficients of the Euler–Lagrange form.

Proposition 2.3. *Let* dim $X = 4$. *Let* $E = E_\nu\, \omega^\nu \wedge \omega_0$ *be a second order Euler–Lagrange form with coefficients* E_ν *satisfying (1), and let* $\bar{\alpha}$ *be a Hamiltonian system (3) admitting the decomposition* $\bar{\alpha} = \alpha_E + \phi$, *then* α_E *is closed.*

Proof. We have $\bar{\alpha} = \alpha_E + \phi$, $d\bar{\alpha} = 0$ and ϕ does not depend upon E. For $E = 0$: $\alpha_E = 0$, yielding $d\phi = 0$. Hence $d\alpha_E = d\bar{\alpha} - d\phi = 0$. This is completes the proof. □

Conclusion

The differential geometry tools are very useful for application to Hamilton (field) theory. The “geometrization” of Euler–Lagrange and Hamilton theory is used e.g. in [1] - [5], [7], [8].

The paper is generalization of classical Hamiltonian field theory on fibred manifold. The regularization procedure and Lepagean equivalent of the first order Lagrangians was proposed by Krupková and Smetanová [5]. The concept of the Lepagean equivalent of the Euler–Lagrange forms was given in [4]. In the paper the case of first order Hamiltonian systems related to an affine second order Euler–Lagrange form is studied. In dimension 4 the structure of Hamiltonian systems (i.e., Lepagean equivalent of an Euler–Lagrange form) is found.

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Hamiltonovy systémy v dimenzi 4

Abstrakt: Článek je věnován Hamiltonovým systémům v dimenzi 4. Je zde studován systém afinní Eulerovy–Lagrangeovy formy druhého řádu. V dimenzi 4 je nalezena struktura Hamiltonových systémů (tj. Lepageových ekvivalentů Eulerovy–Lagrangeovy formy).

Klíčová slova: Eulerova–Lagrangeova forma, Lepageův ekvivalent Eulerovy–Lagrangeovy formy, Hamiltonův systém.