Interactive 3D graphics generated by Asymptote software to support lessons of differential calculus of functions of two variables

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Abstract: The aim of this contribution is to introduce a collection of solved examples which contain interactive 3D graphics to support lessons of differential calculus of functions of two variables. Each example is processed in two versions. The first one consists of examples which are described in details, so they are suitable for self-study. The second includes examples designed for the class lessons, only main steps are shown in this version. Examples were chosen in a way that we were able to show a geometric meaning of all given terms (like partial or directional derivative, local extremum, global extremum, differential of a function, Taylor polynomial, contours of a function) by interactive 3D graphics. Each graphics could be rotated, zoomed, etc. Whole collection is freely available for students and teachers at creator website or at website of the mi21 project of the VŠB – TU Ostrava.

Keywords: interactive 3D graphics, differential calculus, functions of two variables, solved examples

1 Introduction

Imagine situation that we have to explain terms of differential calculus of functions of one variable to students during a lesson. There won't be any difficulties from a point of view of mathematical principles. Moreover, we are able to draw a simple example of geometric meaning of given term, so students could see that situation without any problems. If we move to a world of differential calculus of functions of two variables, situation will be different. Of course, there won't be any difficulties from the point of view of mathematical principles, either, but if we will have to picture given problem, it won't be easy for the most of us. We don't want underestimate anyone of you, someone could be more talented in drawing. Someone could be able to draw an arbitrary geometric meaning of a term of differential calculus of functions of two variables but if someone really could, there won't be available any rotation of a figure or any zoom, etc. That figure has to be redrawn to a new one. Author of this contribution also remember situation

in which a teacher tried to illustrate a tangent plane by a paper and notebook. Notebook represent the tangent plane and the paper a function. It was a brave attempt how to picture the geometric meaning of the problem but not quite effective. However, time has changed and it brings to us a more user friendly technical support.

For our purpose we chose a software called Asymptote which is able to generate interactive 3D graphics, for more information see [1], [2] or some examples [3]. Syntax of this software is based on C++, so any of you, who know the basics of C++, could use the Asymptote without any problems. A big, and we have on mind a really big, advantage of the Asymptote is a fact that we could, and still can, include an Asymptote code into a LaTeX document, so we create a PDF document with interactive 3D graphics as a one whole document. PDF document created by this way has to be opened in PDF viewer which is able to active a multimedia content (e.g. Adobe Reader for Windows OS). Otherwise, you won't be able to activate the interactive 3D graphics which is included in the document. To active it a single left click on a mouse will be sufficient.

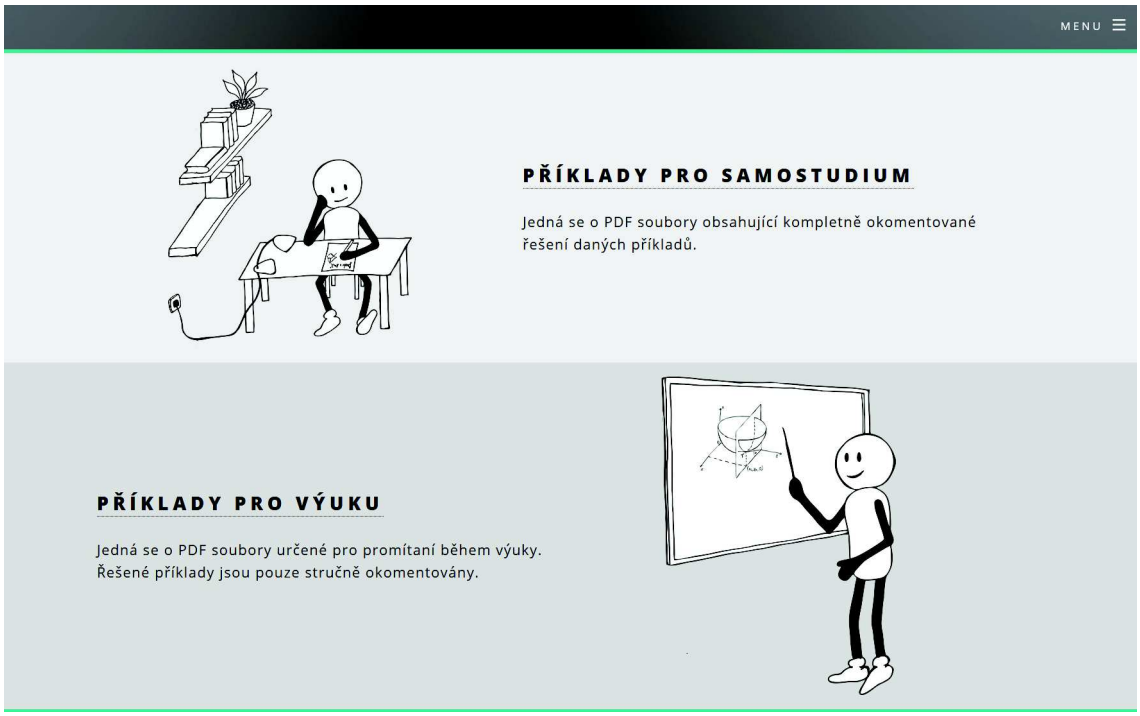

Figure 1: Main menu of the website.

We used the Asymptote to illustrate the geometric meaning of terms of differential calculus of functions consisting of two variables. We created a collection of 24 unique solved examples into which the interactive 3D graphics was added. Examples were chosen in a way that the geometric meaning will be sufficiently illustrative. There are available two versions for each example in the collection. The first one, described in details, is recommended for self-study. The second is suitable for teaching in class lessons, only a necessary steps are shown. Whole collection was uploaded to a creator's website `http://homel.vsb.cz/~fol0037/indexDiff.html/` and to a website of mi21 project `http://mi21.vsb.cz/` of the VŠB – TU Ostrava for better availability from every corner of republic. During creation of the collection we were inspired by [4:76-92], [5] and [6].

A small preview of the collection will be done in the following text.

2 Self-study examples

If you visit a first page of the website `http://homel.vsb.cz/~fol0037/indexDiff.html`, you will find a main menu which is divided into two sections, see Fig. 1. The first one consists of examples designed for self-study ("Příklady pro samostudium" in Czech). The second contains examples suitable for teaching ("Příklady pro výuku" in Czech).

After a click on self-study examples link, the whole collection will appear. It has 8 chapters called: Domain of functions of two variables, contours of functions of two variables, partial derivative, differential of functions of two variables, directional derivative, Taylor polynomial, local extreme and global extreme. In each chapter three examples are included, like you can see in Fig 2.

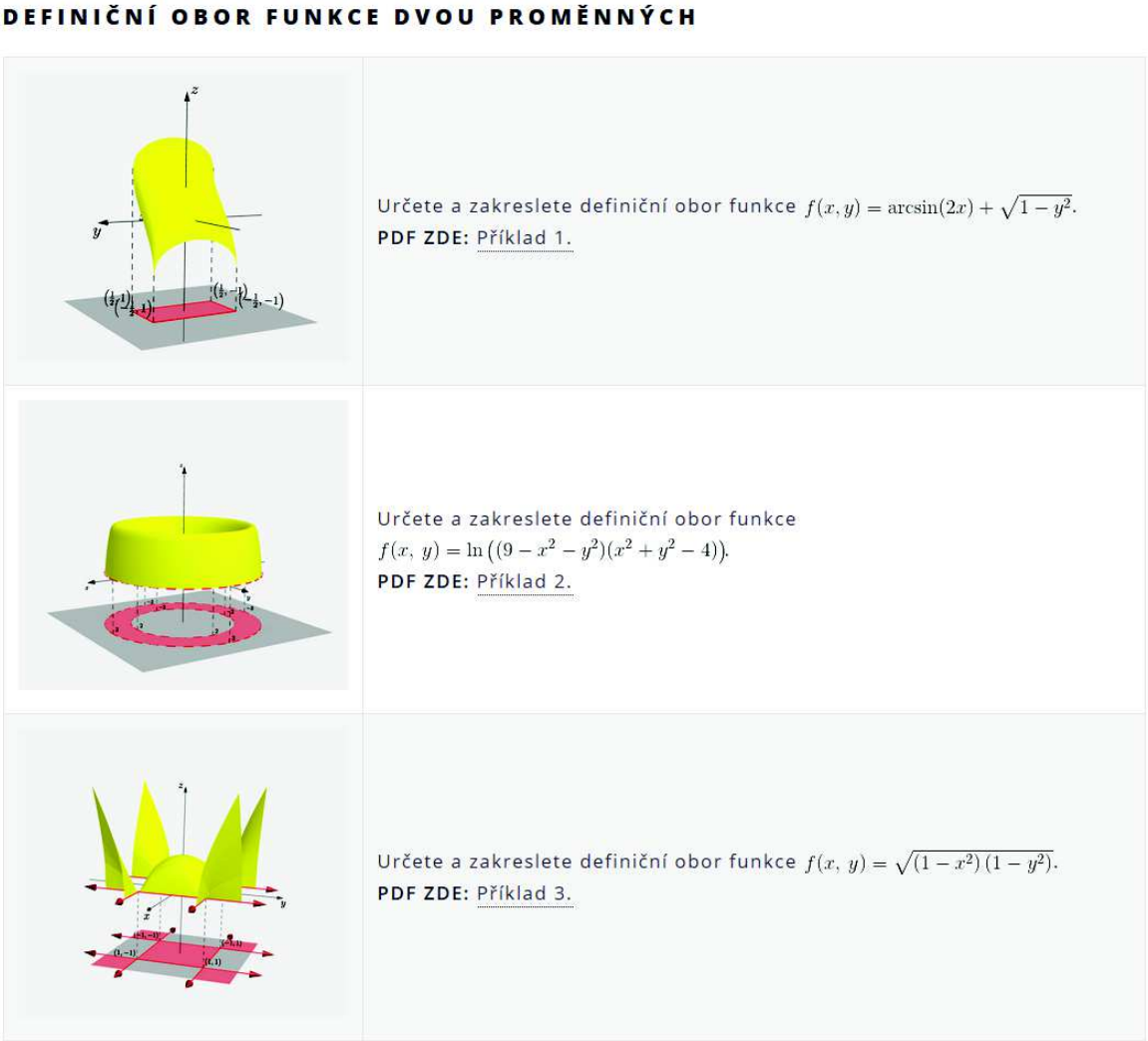

Figure 2: Chapter of domain of function.

There is also available a preview of every example, specifically a task and relevant interactive 3D graphics, within chapters. As it said in a title of this section, these examples were designed for self-study, so they are solved and described in details for easier understanding of given problem, and they were processed in A4 format. We recommend you to visit the website to see

individual examples but we note that they are written only in czech language. Due to this fact we don't attached an example of some self-study document page to this contribution. What we can attached is the interactive 3D graphics, so we added here graphics of partial derivative (Fig. 3) and Taylor polynomial (Fig. 4).

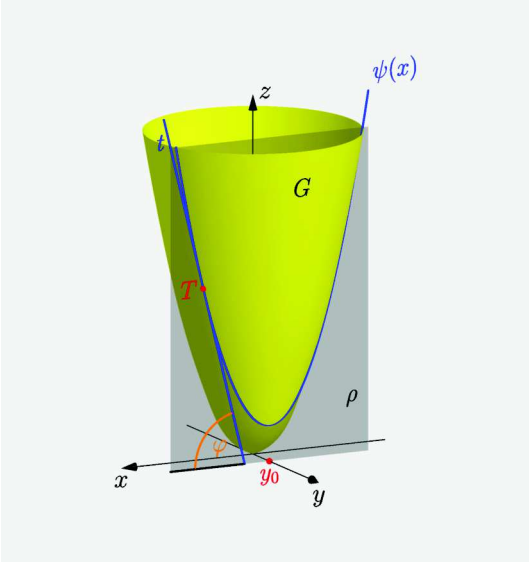

Figure 3: Partial derivative $x^2 + y^2$ by x at $(2; 1)$.

If you have an electronic version of the contribution or you have visited the website, you can activate the graphics by single left click on specific figure but, as we said above, you have to use the PDF viewer which can activate the multimedia content. After the click, the graphics is activated and you can view it in a fullscreen mode by using a menu which will appear after a right click on the graphics. Due to interactivity of the graphics, you are able to rotated it,

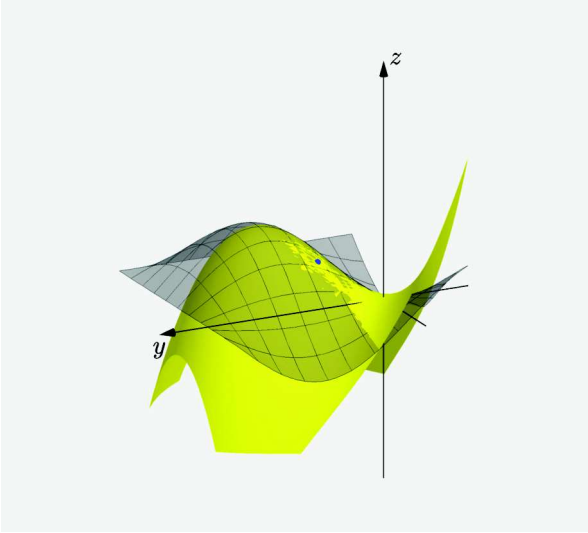

Figure 4: Taylor polynomial of third order of $\sin(x)\sin(y)$ at $(\frac{\pi}{4}; \frac{\pi}{4})$.

zoom it, change a light of scene, etc. Fullscreen preview will be closed by the "escape" key.

3 Examples suitable for teaching

These examples are divided into same chapters as examples for self-study. Also same examples are used so description of them isn't necessary. A difference is in their brevity. In the teaching version only important steps are shown. There isn't any detailed text in this version. We have to emphasize that examples were processed in a format 4:3 which is suitable to be screened during a class lesson. For better idea we added individual slides of a first example of local extremum below. We note that the interactive 3D graphics is also added to this version of examples but we didn't attached it to this example because it is not illustrative here in the contribution as it is in electronic version. The example is written in Czech language but we translated it for purpose of the contribution.

. . . . . .

Local extremum

Example no. 1

Task:

Find local extremum of given function

$$f(x,y) = \frac{27}{10}x^2y + \frac{14}{10}y^3 - \frac{69}{10}y - \frac{54}{10}x \ .$$

. . . . . .

Solution:

Partial derivatives of the first order are equal to

$$\frac{\partial f}{\partial x}(x,y) = \frac{54}{10}xy - \frac{54}{10} \ ,$$
$$\frac{\partial f}{\partial y}(x,y) = \frac{27}{10}x^2 + \frac{42}{10}y^2 - \frac{69}{10} \ .$$

We express variables x and y from a system of two variables (stationary points).

. . . . . .

$$\frac{54}{10}xy - \frac{54}{10} = 0$$
$$\frac{27}{10}x^2 + \frac{42}{10}y^2 - \frac{69}{10} = 0 \ ,$$

$$
\begin{aligned}
54xy - 54 &= 0 & 27x^2 + 42y^2 - 69 &= 0 \\
xy - 1 &= 0 & 27\frac{1}{y^2} + 42y^2 - 69 &= 0 \\
x &= \frac{1}{y}\ , & 42y^4 - 69y^2 + 27 &= 0
\end{aligned}
$$

Substitution: $a = y^2$

$$
\begin{aligned}
42a^2 - 69a + 27 &= 0 \\
a_1 = 1 \quad &\Rightarrow \quad y_1 = 1,\ y_2 = -1 \\
a_2 = \frac{9}{14} \quad &\Rightarrow \quad y_3 = \frac{3}{\sqrt{14}},\ y_4 = -\frac{3}{\sqrt{14}}\ .
\end{aligned}
$$

. . . . . .

We found stationary points $(1,1)$, $(-1,-1)$, $\left(\frac{\sqrt{14}}{3}, \frac{3}{\sqrt{14}}\right)$ and $\left(-\frac{\sqrt{14}}{3}, -\frac{3}{\sqrt{14}}\right)$.

Partial derivatives of the second order are equal to

$$
\begin{aligned}
\frac{\partial^2 f}{\partial x^2}(x,y) &= \frac{54}{10}y\ , \\
\frac{\partial^2 f}{\partial y^2}(x,y) &= \frac{84}{10}y\ , \\
\frac{\partial^2 f}{\partial xy}(x,y) &= \frac{54}{10}x\ .
\end{aligned}
$$

. . . . . .

We assembly a determinant $J(x,y)$

$$
J(x,y) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(x,y) & \frac{\partial^2 f}{\partial xy}(x,y) \\ \frac{\partial^2 f}{\partial xy}(x,y) & \frac{\partial^2 f}{\partial y^2}(x,y) \end{vmatrix}\ .
$$

In our case $J(x,\ y)$ is in a form

$$
J(x,y) = \begin{vmatrix} \frac{54}{10}y & \frac{54}{10}x \\ \frac{54}{10}x & \frac{84}{10}y \end{vmatrix} = \frac{4536y^2 - 2916x^2}{100}\ .
$$

. . . . . .

Points $(1,1)$ and $(-1,-1)$ satisfies

$$
J(x,y) = \frac{1620}{100} > 0\ ,
$$

i.e. there is extremum.

Points $\left(\frac{\sqrt{14}}{3}, \frac{3}{\sqrt{14}}\right)$ and $\left(-\frac{\sqrt{14}}{3}, -\frac{3}{\sqrt{14}}\right)$ satisfies

$$J(x,y) = -\frac{1620}{100} < 0 \ ,$$

i.e. there is no extremum.

. . . . . .

Point $(1,1)$ satisfy

$$\frac{\partial^2 f}{\partial x^2}(1,1) = \frac{54}{10} > 0 \ ,$$

so local minimum is in this point.

Point $(-1,-1)$ satisfy

$$\frac{\partial^2 f}{\partial x^2}(-1,-1) = -\frac{54}{10} < 0 \ ,$$

so there is local maximum in this point.

. . . . . .

As you can see, there are shown only important steps in this example. We presume that a teacher will be able to say some details by yourself.

Conclusion

We introduced you the collection of 24 solved examples which are exceptional because interactive 3D graphics is added to them. Examples are divided into 8 chapters corresponding to terms of differential calculus of functions consisting of two and more variables. We also noticed that the collection was uploaded to the creator's website and website of the mi21 project.

The collection is already used to support lessons of differential calculus at the VŠB – TU Ostrava university for few years. It was created as author's bachelor thesis under leadership of RNDr. P. Vondráková, Ph.D. The collection was lately extended in a frame of internal SGS project in cooperation with Ing. N. Plívová.

In a future work, we think of translation this collection into English and its usage for lessons of foreign students.

References

[1] "Official website of the Asymptote." Internet: `http://asymptote.sourceforge.net`, 2004 [Jan. 16, 2013].

[2] O. Kutal. "Tvorba matematické grafiky pomocí programu Asymptote." Master thesis, Masaryk's university, Czech republic, 2012.

[3] "ASYMPTOTE – Galeries d'examples." Internet: `http://marris.org/asymptote`, 2011 [Jan. 14, 2013].

[4] R. Plch, P. Šarmanová. "Interaktivní 3D grafika v HTML a PDF dokumentech." *Zpravodaj Československého sdružení uživatelů TEXu* (Jan. 2, 2008), p. 76-92.

[5] S. Kuráňová, J. Vondra. "Diferenciální počet funkcí více proměnných – interaktivní sbírka příkladů a testových otázek." Internet: `http://is.muni.cz/elportal/?id=857340`, 2009 [Feb. 10, 2013].

[6] J. Kuben, Š. Mayerová, P. Račková, P. Šarmanová. "Diferenciální počet funkcí více proměnných." Internet: `http://mi21.vsb.cz/modul/diferencialni-pocet-funkci-vice-promennych` [Jan. 15, 2013].

Interaktivní 3D grafika generovaná programem Asymptote pro podporu výuky diferenciálního počtu funkcí dvou proměnných

Abstrakt: Cílem příspěvku je představení sady řešených příkladů s interaktivní 3D grafikou pro podporu výuky diferenciálního počtu funkcí dvou proměnných. Každý z příkladů je k dispozici ve dvou verzích. Podrobnější verze je určená pro samostudium, stručnější pro promítání během výuky. Příklady jsou zvoleny tak, aby bylo možno geometrický význam všech pojmů demonstrovat pomocí interaktivní 3D grafiky. S grafy, které ilustrují pojmy jako parciální a směrové derivace, diferenciál, Taylorův polynom, lokální a globální extrémy a mnoho dalších, je možno interaktivně pracovat (otáčet, přibližovat,...). Sbírka je volně k dispozici studentům i pedagogům pro využití při výuce a to na webových stránkách autora a webových stránkách projektu „Matematiky pro inženýry 21. století" (`mi21.vsb.cz`) VŠB – TU Ostrava.

Klíčová slova: iteraktivní 3D grafika, diferenciální počet, funkce dvou proměnných, řešené příklady