Solids and surfaces of revolution in GeoGebra

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Abstract: We present the application tool for demonstration of the Riemannian definite integral based on decomposition of a given solid onto the elementary cylinders and their surfaces of revolution in GeoGebra. The tool is presented in the form of worksheets with detailed commentary for the reader. This tool may support students' imagination.

Keywords: GeoGebra, Riemannian definite integral, solids of revolution, surfaces of revolution.

1 Introduction

GeoGebra is one of the best free software and supports education process in mathematics and descriptive geometry. GeoGebra also improves students' imagination and simplify mathematical concepts.

We use GeoGebra in many different ways during education of mathematics. Student can meet GeoGebra during lectures, they also work with GeoGebra during tutorials and they use GeoGebra at their homes for elaboration of homeworks and their own preparation for maths.

GeoGebra can cover many important topics, namely: differential and integral calculus of functions depending on one or two real variables; linear algebra - matrices, systems of linear equations, vectors; analytical geometry - lines, planes, conic sections, polygons; solids - spheres, cylinders, cones, pyramids, prisms, Platonic solids; probability theory and statistical methods; geometric mappings and planimetric and stereometric problems.

In the paper we present an applet for demonstration of the volume and the surface of solids of revolution which arise by rotations of graphs of some functions. This applet is in the form of worksheets which enable the reader to follow every single step necessary for the creation of this tool, c.f. [1–4].

2 Concept of the applet

We divide the applet into two parts. The first part called control panel enable the user to input arbitrary function and set its lower and upper limits. Then GeoGebra calculate the

volume of the appropriate solid of revolution which arises by the rotation of area under the graph of the given function around x-axis, Fig. 1.

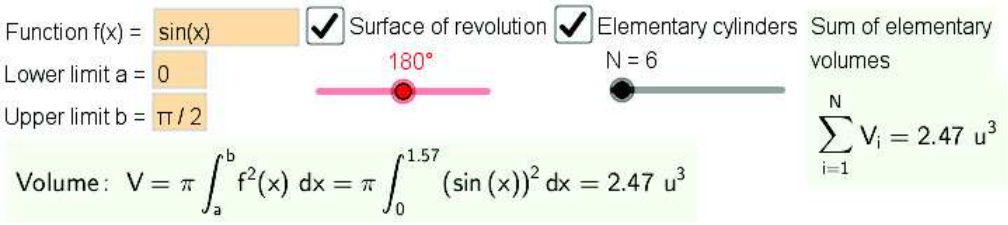

Figure 1: Control panel

The second part of the applet contains visual representation of the problem and a decomposition of the solid onto elementary cylinders, Fig. 2.

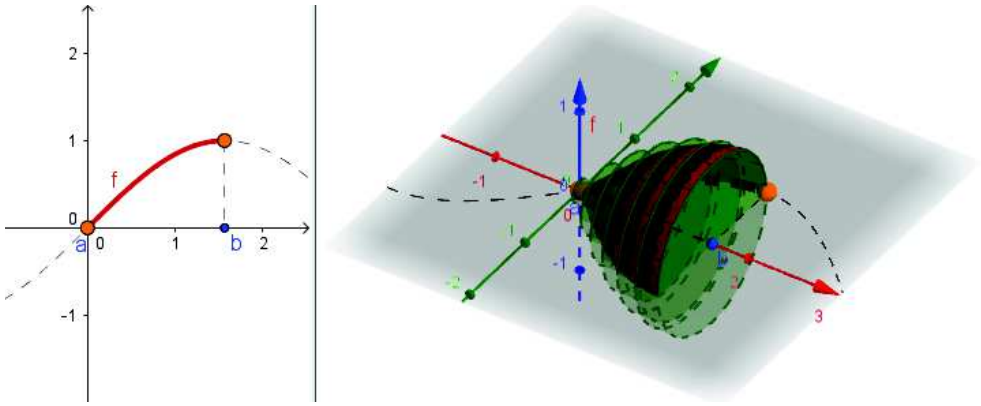

Figure 2: Representation of volume

The orange points can be dynamically changed to set new range of the integration.

3 Worksheets

The following tables contain sequences of commands for GeoGebra applet. At first we open in GeoGebra the following windows: Graphics, Graphics2 and 3D Graphics. We place 2D objects into Graphics, the control panel into Graphics2 and 3D objects into 3D Graphics window.

1.	Input:	Graphics. Create Function `g` by entering `g(x)=sin(x)` into the *Input Bar*, set Line Style to dashed line, Hide Label.
2.	Input:	Create Number `a` by entering `a=0` and create Number `b` by entering `b=π/2` into the *Input Bar*.
3.	Input:	Create bounded function `f` by entering `f(x)=Function[g,a,b]` into the *Input Bar*, change color to red, change Line Thickness to 7.
4.	a = 1	Graphics2. Create *Input Box* with Caption `Function f(x)=` and Linked Object g(x). Similarly create *Input Box* with Caption `Lower limit a=` and Linked Object `a`. and create *Input Box* with Caption `Upper limit b=` and Linked Object `b`

5.		Graphics2. Create *Check Box* `sur` by entering `sur=CheckBox["Surface of revolution"]` into the *Input Bar*. Create *Check Box* `cyl` by entering `cyl=CheckBox["Elementary cylinders"]` into the *Input Bar*.
6.		Graphics2. Insert *Slider* `angle` from 0° to 360° with increment 1°, change color to red, change width to 120 px. Set value to `180°`.
7.		Graphics2. Insert *Slider* `N` from 1 to 100, with increment 1, change width to 120 px. Set value to `6`.
8.		Create auxiliary value `WidthX=(b - a) / N`.
9.		Create auxiliary object `DividingPoints= Sequence[a + i WidthX, i, 0, N]`.
10.		Create auxiliary object `ReprPoints=Sequence[DividingPoints(i) + WidthX / 2, i, 1, N]`.

We use the preceding objects for creation of elementary cylinders. These cylinders decompose given solid of revolution and provide us with an approximation of the volume of the solid. The volume of the solid is also calculated using definite integral to obtain the exact value of the volume. Finally, one can compare both values of the volume, the approximated and the exact ones.

11.		3D Graphics. Create elementary cylinders `ListCylinders= Sequence[Cylinder [(DividingPoints (i), 0, 0), (DividingPoints(i + 1), 0, 0), g(ReprPoints(i))], i, 1, N]`.
12.		Calculate volume of cylinders `VolumeCylinders=Sum[ListCylinders]`.
13.		Calculate volume of solid of revolution `VolumeSolid=Integral[pi f(x)^2, a, b]`. Hide object.
14.		Graphics. Calculate area of region under function `f`, `Areaf=Integral[ f(x), a, b]`. Hide object.
15.		Graphics. Create first point on graph `A=Point[g]`. Open Object Properties, Scripting, On Update and put `SetValue[a,x(A)]`.
16.		Graphics. Create second point on graph `B=Point[g]`. Open its Object Properties, Scripting, On Update and put `SetValue[b,x(B)]`.
17.		Open Object Properties of `InputBox1`, set Scripting, On Update, put commands `SetActiveView[1]; CenterView[((a+b)/2,Areaf/(2*(a+b)))];`.
18.		Open Object Properties of `InputBox2`, set Scripting, On Update by commands `SetActiveView[1]; CenterView[((a+b)/2,Areaf/(2*(a+b)))]; SetValue[A, (a,g(a))];`.
19.		Open Object Properties of `InputBox3`, set Scripting, On Update by commands `SetActiveView[1]; CenterView[((a+b)/2,Areaf/(2*(a+b)))]; SetValue[B, (b,g(b))];`.
20.		Graphics. Create auxiliary point `AuxA=(x(A),0)` on x-axis and put Caption "a".
21.		Graphics. Create auxiliary point `AuxB=(x(B),0)` on x-axis and put Caption "b".
22.		Graphics. Create auxiliary segment `Segm1=Segment[AuxA,A]` open Object Properties, Hide Label and set dashed line.
23.		Graphics. Create auxiliary segment `Segm2=Segment[AuxB,B]` open Object Properties, Hide Label and set dashed line.
24.		Graphics2. Create new text `text1` containing "Sum of elementary volumes".

25.		Graphics2. Create new text `text2` containing `"\sum_{i=1}^N V_i= VolumeCylinders u^3"`, and mark LaTeX. The highlighted object `VolumeCylinders` must be selected from the list of Objects.
26.		Graphics2. Create new text `text3` containing `"\mbox{Volume\!: }V=\pi\int_a^b \!\!\! f^2(x) \ dx=\pi\int_{a}^{b}\!\!\!\left(g\right)^2 dx= VolumeSolid u^3"`, and mark LaTeX. The highlighted objects `a`, `b`, `g` and `VolumeSolid` must be selected from the list of Objects.
27.		3DGraphics. Create surface of revolution `RotSurface=Surface[f, angle, xAxis]`.
28.		For objects `RotSurface` and `angle` open Object Properties, Advanced, Condition to Show Objects and put `sur`.
29.		For objects `ListCylinders`, `N`, `text1` and `text2` open Object Properties, Advanced, Condition to Show Objects and put `cyl`.

Every user can individually manage position of graphics windows, colors, styles, etc. Note that we use two 2D graphics views, namely Graphics and Graphics2. It is necessary to click into appropriate window if you want to create an object in concrete graphics view. If you cannot see an object you have created, it is possible that the object is placed in wrong graphics view. One can change visibility or the location of an object in Object Properties, Advanced, Location.

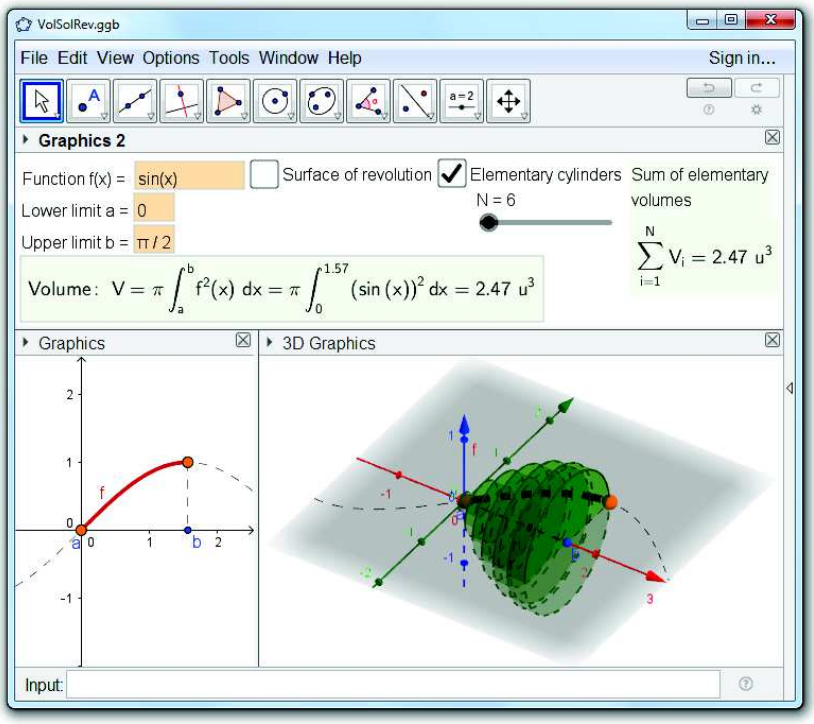

Figure 3: Applet

4 Conclusion

The applet for demonstration of the decomposition of a given solid of revolution onto elementary cylinders can help students to gain better understanding of integral calculus of functions depending on one real variable. The applet can be used during the lecture of mathematics but it is also available for students. GeoGebra helps us to improve education of mathematics. Creation of such applets is one of the activities of our GeoGebra Institute of Ostrava.

Acknowledgments

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Rotační tělesa a jejich pláště v GeoGebře

Abstrakt: Prezentujeme nástroj pro demonstraci Riemannova určitého integrálu založený na rozkladu daného rotačního tělesa na elementární válce v GeoGebře. Nástroj je prezentován ve formě pracovních listů s detailním komentářem pro čtenáře. Tento nástroj by mohl sloužit k podpoře rozvoje představivosti studentů.

Klíčová slova: GeoGebra, Riemannův určitý integrál, rotační těleso, plášť rotačního tělesa.